Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 197 JUNE, 1924 No. 6

PRESSURE, SUPERHEAT, STEAM EXTRACTION AND REHEATING AS AFFECTING POWER PLANT ECONOMY.*

BY

ESKIL BERG, E.E., M.E.

Consulting Engineering Department, General Electric Company, Schenectady, N. Y.

DUE to the rapid and ever-increasing price of fuel, central power stations have, during the last year or two, done more towards obtaining station economy than at any previous time.

The adoption of the steam turbine many years ago practically revolutionized central station practice. This prime mover has now, however, reached a point of development where its efficiency is within a few per cent. of that theoretically available, so that further improvement in the turbine cannot materially reduce the fuel consumption. If radical improvements are to be made we must look toward the steam condition itself.

The more energy (B.t.u.) we can supply to each pound of steam when it enters the turbine, the less number of pounds will naturally be required to produce the unit of power (kw. hours). To supply more energy to a pound of steam means to raise its temperature. This can be done by either raising the steam pressure or adding superheat, or doing both. Steam can also be

* Presented at the Stated Meeting of the Institute held Wednesday, March 19, 1924.

(Note.—The Franklin Institute is not responsible for the statements and opinions advanced by contributors to the JOURNAL.)

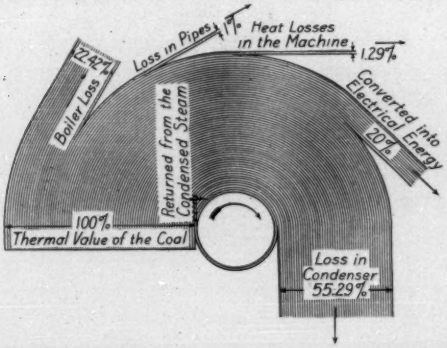
COPYRIGHT, 1924, by THE FRANKLIN INSTITUTE.

extracted from the turbine at some lower pressure, then reheated and led back to the turbine.

Fig. 1 represents graphically how the energy of a pound of coal is distributed in a first-class power station using 250-pound pressure; 200 degrees superheat and under good vacuum conditions.

This shows that 20 per cent. of the energy in the coal is converted into electrical energy, which figure corresponds to

FIG. 1.


Loss in Pipes 1%
Heat Losses in the Machine 1.29%
Converted into Electrical Energy 20%
Boiler Loss 22.42%
Returned from the Condensed Steam
100% Thermal Value of the Coal
Loss in Condenser 55.29%

17,075 B.t.u. per kilowatt hour, which is about the lowest figure at which power has been produced under the above steam conditions.

It will also be noted that the condenser loss is by far the largest loss, being 55.29 per cent. of the heat value of the coal. With this picture in view, it will easily be understood that the better the steam consumption is per kilowatt hour, the less steam goes to the condenser. Consequently the condenser loss is reduced in the same proportion.

It was only a few years ago that it was considered good to be able to produce a kilowatt hour with 15 pounds of steam going to the condenser. To-day, with the adoption of high steam pressure and superheat, reheat and bleeding the turbine for heating the feed water, this figure has been more than cut in half, which means that the condenser loss is practically cut in half.

GAIN BY HIGH STEAM PRESSURE.

The total heat of steam gradually increases with the pressure until it reaches its maximum at 370 pounds absolute and remains constant up to 395 pounds absolute, after which the total heat decreases. At 1200 pounds pressure the total heat is only 1179.7, which is the same as at 62 pounds absolute. (Goodenough's steam table.)

Table I shows the theoretical gain by raising the steam pressure. Two hundred pounds has been considered the starting-point and the available energy is given when one pound of steam

TABLE I.

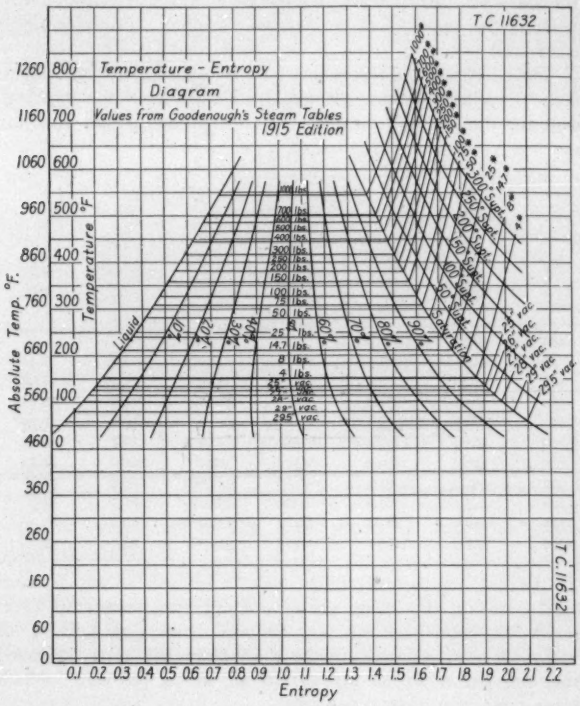
Absolute Pressure Pounds.	Corresponding Temperature–Deg. F.	Total Heat.	Increase in Total Heat. (Per Cent.)	Available Energy in ft. lb. Expanding to 28.5 in Vacuum.	Increase of Available Energy. (Per Cent.)	Net gain in Fuel. (Per Cent.)
200	381.9	1198.5	...	272,000	....	....
300	417.5	1201.9	0.28	293,000	7.72	7.44
400	444.8	1202.5	0.33	304,500	11.95	11.62
500	467.2	1201.7	0.27	312,000	14.7	14.43
600	486.5	1199.8	0.11	319,000	17.2	17.09
700	503.4	1197.4	0.902	323,000	18.7	18.79
800	518.5	1194.4	−0.342	327,000	20.2	20.54
900	532.3	1191.1	−0.617	329,000	20.9	21.51
1000	545.0	1187.6	−0.909	331,000	21.7	22.6
1100	556.6	1183.8	−1.225	335,000	23.2	24.42
1200	567.7	1179.7	−1.58	337,000	24.0	25.58

is expanding adiabatically down 28.5″ vacuum. It will be seen that by raising the steam pressure from 200 pounds to 1200 pounds, the total heat has been reduced from 1198.5 B.t.u. to 1179.7, or 1.58 per cent., while the energy has been increased from 272,000 ft. lb. to 337,000 ft. lb., or 24 per cent., which means a net gain of 25.5 per cent.

The gain by high steam pressure also naturally increases the output of a given turbine because the length of the bucket of the last row limits the output of the turbine. It is found that, if the steam conditions are changed from 250-lb. gage—200° superheat—29″ vacuum to 500-lb gage—725° temperature—29″ vacuum, the leaving loss of the steam is reduced to 63 per cent.

of what it would have been on a machine of the same capacity and bucket dimensions designed for 250-lb. pressure. A reduction to 63 per cent. in the loss of the last bucket means 26 per cent.

FIG. 2.



more capacity for the same loss. This does not, however, mean that a less expensive turbine can be built when adopting high steam pressure, as naturally a turbine designed for high steam pressure will have more stages and be built of special material, making it slightly more expensive than when designed for ordinary pressure.

HIGH SUPERHEAT.

The theoretical gain by the use of superheat is almost negligible as will be seen by Table II. There is, however, a practical gain in its adoption which makes superheat necessary. It is clear by looking at the entropy diagram (Fig. 2) that the moisture in the exhaust increases as the initial pressure is increased, and with 1000-lb. pressure and 200° superheat the moisture in the steam, when expanding down to 29", is about the same as when

TABLE II.

Two Hundred and Fifty Pounds, Twenty-eight and One-half Inches, Superheat.

Deg. Superheat.	Temp.	Avail. Heat.	Corresponding ft. lb.	Total Heat.	Per Cent. Increase in Fuel to Produce Superheat.	Per Cent. Increase Available Energy Due to Superheat.	Net Theo. Gain. (Per Cent.)	Decrease in Water Rate. (Per Cent.)	Actual Gain. (Per Cent.)
0	406	369.9	288,000	1201.1	...	...	...	..	...
50	456	381.4	297,000	1233	2.66	3.11	.45	4	1.34
100	506	393.5	306,000	1264	5.25	6.38	1.14	8	2.75
150	556	405.9	316,000	1292	7.57	9.74	2.17	12	4.43
200	606	418.7	326,000	1320	9.90	13.22	3.32	16	6.10
250	656	431.8	336,500	1346	12.08	16.75	4.67	20	7.92
300	706	445.3	347,000	1374	14.40	20.4	6.00	24	9.6

200-lb. pressure, dry steam, is expanded to 29". Moisture in a turbine affects its efficiency very much and is its greatest source of loss. The losses in an average, well-designed turbine of the impulse type, when dry initial steam is used, are of the following magnitude:

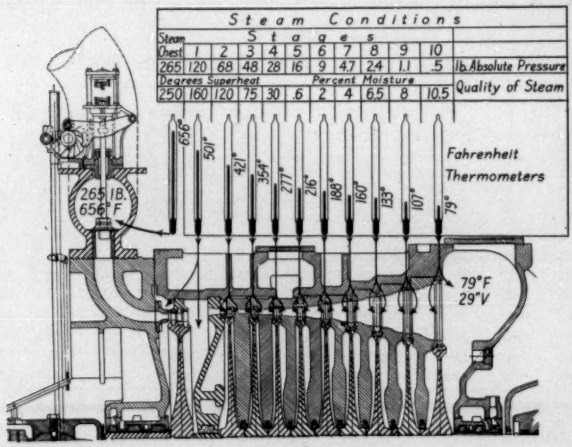
	Per Cent.
Loss due to friction in nozzles and blades and windage loss of discs and blades	20
Steam leakage loss	3
Rejected energy (due to residual steam velocity)	3
Bearings, packings, etc.	1
Total loss	27
Efficiency of turbine	73

It will be seen that the first item is by far the largest one and the moisture in the steam in all the stages has greatly increased this loss. If, on the other hand, steam instead of being dry had entered the turbine with 200° of superheat, it would be found that

this loss would be reduced from 20 per cent. to about 15 per cent., and the turbine efficiency would be increased from 73 per cent. to 78 per cent.

Fig. 3 shows a cross-section of a turbine having ten stages and gives approximately the condition of the steam in all the stages. This turbine has 80 per cent. efficiency and is supplied

FIG. 3.


Steam Conditions
Steam
Stages
Chest 1 2 3 4 5 6 7 8 9 10
265 120 68 48 28 16 9 4.7 2.4 1.1 .5 lb. Absolute Pressure
Degrees Superheat
Percent Moisture
250 160 120 75 30 .6 2 4 6.5 8 10.5 Quality of Steam
656° 501° 421° 354° 277° 216° 188° 160° 133° 107° 79°
Fahrenheit Thermometers
265 lb. 656° F
79° F 29" V

with steam at 250-lb. gage pressure—250° superheat, and is exhausting into 29" of vacuum.

Table III gives a comparison of the condition of the steam when supplied with 250° superheated steam and when supplied with initial dry steam.

TABLE III.

Table Comparing Quality of Steam in Various Stages, when Dry Initial Steam and Steam Having 250° of Superheat Is Used.

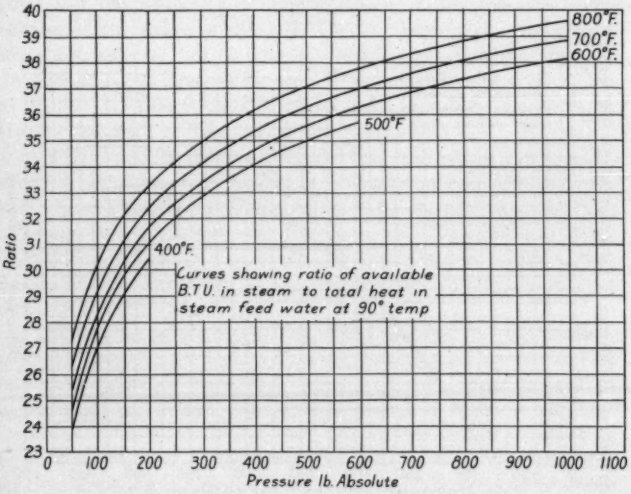
Stages	1	2	3	4	5	6	7	8	9	10
Quality of steam	Per cent. moisture									
Dry initial	4.4	6.8	8.15	9.8	11.5	13	14.5	16	17.3	18.7
Quality of steam	Degrees superheat					Per cent. moisture				
250° superheat	160	120	75	30	.6	2	4	6.5	8	18.5

GAIN BY HIGH STEAM PRESSURE AND SUPERHEAT COMBINED.

Fig. 4 gives the ratio of available B.t.u. in steam to the total heat in steam, giving credit for the heat in the condensate which is returned at a temperature of 90° F.

Up to the present time the standard steam pressure for large power stations has been 250 pounds with superheat of about 200°

FIG. 4.



and 28.5" vacuum. From the curve this gives a ratio of 32.8 per cent.

At the present time several large stations are building in this country which have adopted 550 lb.—725°. From the curve this gives a ratio of 36.9 per cent. With the same turbine and boiler efficiency this would mean a saving of 12.2 per cent.

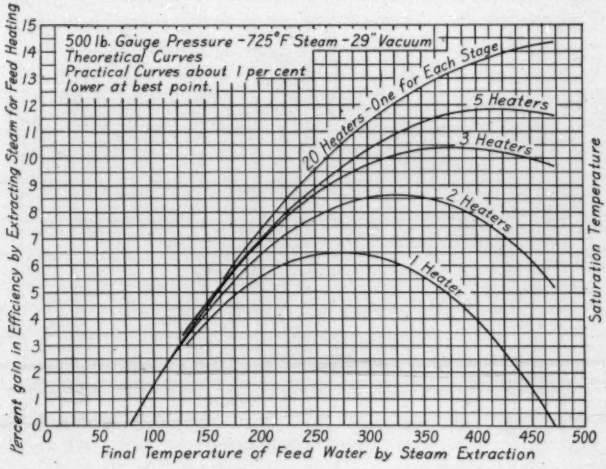
The ratio 36.9 would give a kilowatt hour, assuming a turbine efficiency of 75 per cent. and boiler efficiency of 80 per cent., with $36.9 \times .75 \times .80 = 22.2$. As one kilowatt hour = 3415 B.t.u., this corresponds to $\frac{3415}{222} = 15{,}350$ B.t.u., to which has to be added about 5 per cent. for station auxiliaries, making it = 16,100 B.t.u.

STEAM EXTRACTION.

A very large gain in efficiency is made in the cycle shown in Fig. 4 by extracting steam for heating the feed water from several stages in the turbine.

Fig. 5 shows approximately the theoretical gain by the use of 1, 2, 3, 5 and 20 heaters. While these curves are calculated for 500-lb. gage—700° and 29", they can be used with compara-

FIG. 5.


500 lb. Gauge Pressure -725°F Steam -29" Vacuum
Theoretical Curves
Practical Curves about 1 per cent lower at best point.
20 Heaters - One for Each Stage
5 Heaters
3 Heaters
2 Heaters
1 Heater
Percent gain in Efficiency by Extracting Steam for Feed Heating
Saturation Temperature
Final Temperature of Feed Water by Steam Extraction
0 50 100 150 200 250 300 350 400 450 500
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

tively small error for other pressures and temperatures. If the steam is initially dry, the gain would be larger than what is shown but only by a couple of per cent.

Extraction of steam for heating feed water has also another important gain, namely, the more steam we extract from the turbine before it reaches the last row of blades, the less steam goes through the last row of blades, and consequently reduces the congestion, so that, for instance, 20 per cent. extraction of steam reduces leaving losses to 63 per cent. of what it would be in per cent. of available energy for the same capacity and same last stage wheel.

REHEATING.

From a glance at the entropy diagram given in Fig. 2 it will be seen that, when adopting pressure of 500 and above, there will be excessive moisture in the low-pressure end of the turbine. Reheating is, therefore, more advisable under those steam conditions than when lower steam pressure is used, although at the lower pressure it is also advisable. But with low initial pressure (250 lb.) the reheating is done at about 50 lb. pressure. The volume of the steam at this pressure is rather large, requiring a large size reheater not to have too much drop in steam pressure through the reheater. The most economical pressure at which to do the reheating varies but, as a rule, it will be found to be around one-fifth of the initial boiler pressure.

Theoretically, as with initial superheat, the gain is small, being about 3 per cent. if the steam is extracted at one-fifth pressure and reheated to original temperature. The decrease in water rate, however, is about 4 per cent. due to reduction of friction losses in turbine. There is, nevertheless, a slight loss in pressure, etc., due to the reheater itself, so that the net gain will be about 6 per cent. in efficiency.

The leaving losses of the last wheel are also reduced because, as more heat energy is added to the steam when reëntering the turbine, less steam naturally is required to produce a unit of power (kilowatt hour) which means that less steam reaches the last row of buckets. It is found, for instance, that, if reheating is done up to 700° F., which is the initial total temperature, the leaving losses of the steam in the last stage are reduced to about 80 per cent. of what it would have been without such reheating.

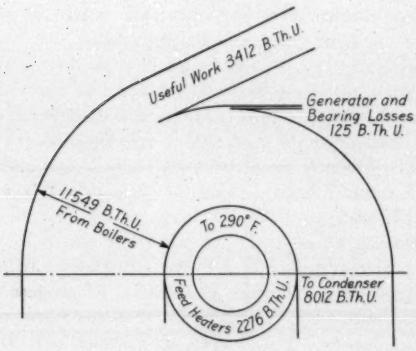
ECONOMIZER.

Steam extraction from the turbine for heating the feed water is so efficient that the use of economizers would seem unnecessary. However, the flue gas temperature must be kept low in order to keep the efficiency of the boiler plant high. Thus, if economizers are omitted, the waste heat from the gases must be used to preheat the air going to the boilers. Grate manufacturers at the present time prefer to limit the temperature of the air to 300° F. or below. However, with oil or powdered coal, high air temperature has proved rather beneficial.

CAPACITY RATING.

From the above it will be seen that by adopting 500-lb. gage pressure, 700° F. total temperature, reheating at 100 lb. to 700° temperature, and having 20 per cent. steam extraction for heating feed water, instead of the usual 250-lb. pressure—200° superheat—29" vacuum, the leaving loss of the last bucket has been

FIG. 6.


50,000 Kw. 1800 R.P.M Turbine
350 lb. Gauge 725°F. Temp. 29" Vac.
Heat Distribution
per Kilowatt Generated
Useful Work 3412 B.Th.U.
Generator and Bearing Losses 125 B.Th.U.
11549 B.Th.U. From Boilers
To 290° F.
Feed Heaters 2276 B.Th.U.
To Condenser 8012 B.Th.U.

$$\text{Efficiency} = \frac{3412}{11549} = 29.54 \text{ Per cent}$$

For Fuel Consumption, Efficiency of Boiler must be allowed for, also Auxiliaries

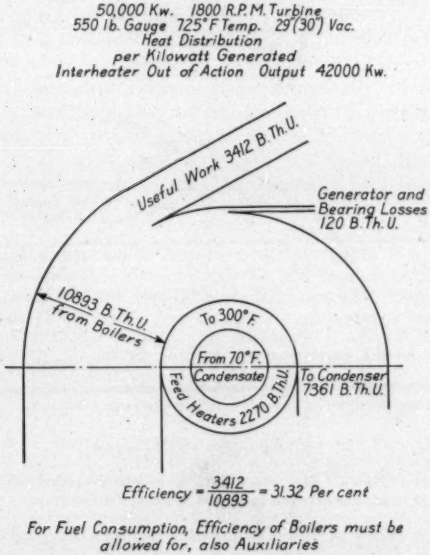
reduced to 63 per cent. on account of high pressure, 63 per cent. due to steam extraction and 80 per cent. due to reheating, making a total reduction of 31.8 per cent. As the capacity is inversely proportionate with the square root of the above, the capacity rating of the turbine has been increased 1.78 times as much. This means also that the percentage loss due to steam friction has been reduced approximately in the ratio of 1 to 1.78, so that a more efficient turbine can be built at this larger rating.

The above article gives the theoretical and practical advantages obtained by the adoption of high steam pressure, high super-

heat, reheating and steam extraction. However, what the central station operator is interested in is the number of heat units required in the turbine to produce a kilowatt hour at the switchboard.

Fig. 6 gives typical heat balance of a tentative turbine designed for 50,000 kw. and 1800 R.P.M. when operating with 350-lb.

FIG. 7.

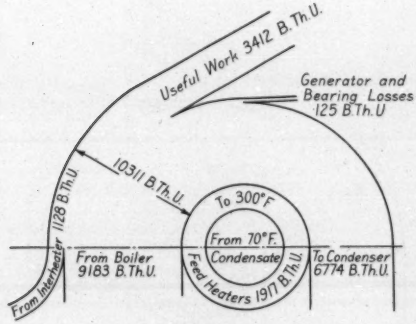


gage pressure—725° F. temperature steam and 29" vacuum, with no reheat but with steam extraction to heat the feed water to 290° F. It shows that one kilowatt hour may be produced with 11,549 B.t.u. from the boilers. For fuel consumption the efficiency of the boilers and power taken by auxiliaries must be allowed for. Fig. 7 gives the heat balance of the same turbine designed to operate with 550-lb. gage—725° F. steam temperature and 29" vacuum. Steam is extracted to heat the feed water to

300° F. With this arrangement a kilowatt hour may be obtained with 10,893 B.t.u. from the boilers.

Fig. 8 gives heat balance of the same turbine designed for reheating the steam to 725° at 100-lb. gage, and gives one kilowatt hour with 10,311 B.t.u. This figure, assuming 80 per cent. boiler efficiency and 6 per cent. for auxiliaries, would mean 13,700

FIG. 8.



B.t.u. from fuel. With oil or powdered coal where full advantage can be taken of air preheating, the boiler efficiency may reach as high as 90 per cent. in which case the fuel consumption would compare favorably with Diesel engines.

HIGH PRESSURE AND HIGH STEAM TEMPERATURE.

Recent turbo-generators on order and building by the General Electric Company:

One 60,000 kw. cross-compound turbo-generating unit for Commonwealth Edison Company, Chicago. This unit is designed for 600-lb. pressure—700° superheat and 29" of vacuum. The

unit consists of one high-pressure 17,000 kw. unit running at 1800 R.P.M. from which the steam is reheated to 700° at approximately 100-lb. pressure and led to the 43,000 kw. low-pressure unit running at 1200 R.P.M.

The American Gas and Electric Company has on order eight (8) 35,000 kw.—1800 R.P.M. units to be installed in three different large power plants. These units are designed to operate at from 600 to 550-lb. pressure—725° steam and 29″ vacuum. Reheating up to 725° is done at 100-lb. pressure.

Two (2) 40,000 kw.—1800 R.P.M. units are also on order for the Columbia Gas and Electric Company, Cincinnati, Ohio.

One (1) 3000 kw. turbine is also being built for the Edison Electric Illuminating Company of Boston to be installed in their new Weymouth Station. This turbine will operate at 1200-lb. pressure—725° steam, exhausting into the 350-lb. station main, and is designed to handle about 130,000 pounds of steam per hour.

Chemical attraction is discussed by DAVID W. HORN (*Trans. Wagner Free Inst. Sci.*, 1923, **10**, 63–71). In exothermic compounds, temperature and stability bear an inverse ratio to each other. Relative temperatures of dissociation of comparable compounds bear a direct relation to the relative strengths of the chemical attraction, or of the resultant of all the attractive and repulsive forces at work within each of the compounds considered. In monovariant systems, corresponding curves may be compared throughout their lengths. When a set of curves representing comparable dissociating systems are compared along a line of constant temperature, and the systems are arranged in the order of the magnitude of the pressures within each at this common temperature, they are thereby arranged in the order of their relative stabilities, the least stable system exhibiting the highest pressure and *vice versa*. When these curves are compared along a line of constant pressure, and the systems are arranged in the order of the temperatures within each at this common pressure, they are thereby arranged in the order of their relative stability, the least stable system exhibiting the lowest temperature, and *vice versa*. At the upper end of the dissociation curve, a temperature may be reached above which it will be impossible to obtain or retain the compound that has been dissociating. Dissociation occurs only at the surface of the solid and the regions contiguous thereto. New data and curves are presented for the thermal dissociation of cuprammonium nitrate and of sodium bicarbonate. J. S. H.

Demonstration of Reproduction of Vowel Sounds. SIR R. PAGET. (*Proc. Phys. Soc.*, Dec. 15, 1923.)—A much more extended account of this work is to be found in the *Transactions of the Royal Society A* (Vol. 102, 1923). "Observation of the resonances of the demonstrator's own vocal cavity, when whispering or breathing the various English vowel sounds, had shown that, in the case of every vowel, the vocal cavity produces two audible resonances, due to the passage of air through the cavity as a whole." The two components are not definitely fixed in pitch. Each may vary a few semitones without introducing any considerable difference in the character of the vowel. A table is given for fourteen vowels setting forth the two components and their possible ranges in pitch. The greatest interval between the pitches of the components is found in the case of the vowel sound of "eat." The lower has a frequency of about 342 and may vary over three semitones, while the upper is near a frequency of 2434 and may have a range of but two semitones. In most of the vowels the lower component is permitted a greater variation of pitch than the upper one, though in the vowel of "who" the reverse is true. The components are nearest together in pitch in the vowel of "not." "Experiments with plasticine models showed that the vocal cavity behaved substantially like two Helmholtz resonators joined together in series. In this arrangement the human tongue acted as a variable and longitudinally moving stop, so as to divide the vocal cavity as a whole into two resonators, each of the appropriate pitch. In the demonstration the human larynx was replaced by a reed pipe, a motor horn or a vibrating strip of rubber. The speaker placed his hands so as to form a double resonator, passed through them the sound from an artificial larynx and by manipulating his hands and fingers produced audibly such sentences as "How are you?" and "Oh! I love London." G. F. S.

Oxygen-enriched Air for Roasting Sulphide Ores.—MESSRS. O'HARRA, U. S. Bureau of Mines, and KAHLBAUM, WHEELER and DARBY, of the Missouri School of Mines, contributed to the February, 1924, meeting of the American Institute of Mining and Metallurgical Engineers, a paper giving data and results of experiments on the action of air containing added oxygen in roasting zinc ores. Their results may have a wider application than the particular treatment which they report. They found that the enriched air increased the amount of sulphur dioxide produced and diminished the time required for roasting. The improvement facilitates the employment of zinc sulphide as a source of sulphuric acid. Suggestions are made for the improvement of the furnaces in which the roasting is done. The wider interest in the matter is that other sulphur ores may be found to be susceptible of satisfactory treatment by enriched air. If the large-scale production of nitrogen and hydrogen is developed in connection with the synthetic ammonia processes, oxygen will be a by-product and its use may make considerable modifications in all classes of furnace operations. H. L.

HYDROGEN-ION CONCENTRATION IN RELATION TO ANIMAL AND PLANT GROWTH.*

BY

HENRY LEFFMANN, A.M., M.D.

Philadelphia College of Pharmacy and Science, Member of the Institute.

SOURNESS is so characteristic a taste, that it must have been recognized as a specific property of some common substances at a very early period of human history. In modern science it is especially noted as a characteristic property of a class of bodies termed "acids." No instance of distinct sourness unassociated with some degree of acidity seems to be known, but compounds are known which have the main characteristics of acid, yet are not appreciably sour. Sourness is not the only noticeable property of acids. Power to dissolve substances insoluble in ordinary liquids is also a striking feature of many. In ancient times but one acid seems to have been recognized as a specific substance, namely, acetic. This exists in moderate proportion in vinegar, and is the result of the exposure of dilute alcoholic liquids to air, by which certain forms of minute organisms are introduced and the alcohol is oxidized to a limited extent. As the part played by these minute organisms has only been recognized within recent years, the change of alcohol to acetic acid was long regarded as "spontaneous" and was so termed. The same erroneous view was held regarding the fermentation of amylaceous and saccharine substances into alcohol, and the putrefaction of organic matters generally.

A review of the early history of acidity and of the discovery of the important types of acids is to be found in the great chemical classic, Hermann Kopp's "Geschichte der Chemie," in the third volume, under the title "Entwicklung der Kenntnisse über die Säure" (Development of knowledge concerning acids). Acetic acid was designated in both Greek and Latin by words akin to those indicating sourness—Greek, *oxos,* vinegar, *oxus,* sour; Latin, *acetum,* vinegar, *acidus,* sour. Dioscorides (first century) gives no specific information, but Pliny the elder states

* Abstract of a paper presented at a meeting held Thursday, December 13, 1923.

that vinegar produces foaming with certain earths. Of course, natural carbonates are meant. Vinegar derived from both wine and cider was known to the Jews, and a sentence in Proverbs (25 : 20) refers to the foaming produced by one of these forms. Kopp gives Luther's translation of the Hebrew text—"Essig auf der Kreide"—vinegar on chalk, but the modern critics read "soda," sodium carbonate being intended. The Hebrew word has the consonants "NTR," which probably was the cause of the King James Bible reading "nitre." The knowledge of the solvent action of vinegar on some familiar substances led to extravagant claims for its power. Ancient writers, including Livy and Plutarch, state that Hannibal in his journey over the Alps used vinegar to dissolve the rocks, and Vitravius claims that it can dissolve rocks that are not affected by fire or iron.

Recognition of the power of vinegar to cause not only foaming, but to form new compounds was a step forward in the differentiation of the group, a conception that was extended when other acids were discovered. It is not necessary to set forth here in detail what might be termed the "lore of acidity," but a few salient points will not be inopportune. Kopp credits Geber with the first reference to nitric acid, called "parting acid," because used in the parting of gold and silver—dissolving the silver out from alloys of the two metals—but Berthelot has thrown considerable doubt on the genuineness of some of the works ascribed to Geber, so that the reference must be used with caution. The term "parting acid," however, persisted until a comparatively recent period as a trade name for a certain strength. The Greek alchemists of the early centuries of the present era added considerably to the list of acids and to a knowledge of their properties. In a much later period Basil Valentine (fifteenth century) discovered hydrochloric acid and improved the method of making sulphuric acid. Naturally, the possession of these three powerful acids must have aided greatly in the development of chemistry. Organic acids, as a group, were recognized about the beginning of the seventeenth century, when Mayerne discovered benzoic acid and Croll succinic. About a century later, Hiärne's observations on formic acid attracted notice as an acid of animal origin. Scheele showed the existence of acids derived from metals, and Boyle pointed out the action on certain vegetable colors with the contrary action of alkaline substances. In 1723, Fr. Hoffmann classified the gas

escaping from some spring waters as an acid, because of its action on litmus. This was carbonic acid.

In the latter half of the eighteenth century, when the civilized world was wrestling with great wars, and especially with the revolt of the American colonies, chemists were investigating the relation of acids to the substances that were capable of saturating them, and a large amount of quantitative data was obtained. The identification of oxygen was an important step forward, and in the laboratory of Lavoisier its relation to acidity was considerably developed. In 1781, Lavoisier presented a memoir to the Paris Academy in which he stated that in the future he would term the gas that he had found to produce acids with some substances and bases with others, "oxygen" (producer of acid), and since that time the word has come into most languages with only slightly modified forms. German, however, in accordance with its system has translated the word into its own roots and calls it "Sauerstoff."

Lavoisier burned sulphur and carbonaceous materials in oxygen and obtained solutions which reddened vegetable colors and neutralized metallic oxides. He assumed, therefore, that the acidity thus produced was due to the gas that he was using and hence the name he gave it. He overlooked the fact that water was present in all these experiments, and we now know what an important part it plays in determining acidity. The discovery that the familiar "muriatic" acid was not a compound of oxygen showed that the part played by that element in determining acidity was less than at first supposed. The acid, together with its allies from the other members of the halogen group, were designated as "hydro-acids," a term still in vogue, through the prefix "hydro" in their names.

As the knowledge of structural composition of compounds developed, it became evident that it is not either hydrogen or oxygen that determines acidity but a particular association of these elements with reference to each other as well as to the rest of the molecule. During the middle period of the last century the electrochemical theories of Berzelius dominated chemistry very largely, and the vast majority of reactions were explained on the principle of attractions by positive and negative elements. Faraday's work contributed very much to the extension of these views. Later, the electrochemical theories fell into some neglect

and were even mildly disapproved, but at present they have come into active application. The introduction and wide acceptance of the ionization theory has contributed greatly to this shift of opinion and out of this theory and the researches that it has initiated, has accrued much information of value in biologic chemistry.

Hydrogen is the element of lowest atomic weight so far known. Although not at present taken as the unit for atomic weights—oxygen having some practical advantages in that respect—its atomic weight may for general purposes be considered as unity. In the modern conception of the atom it is regarded as containing a positive nucleus which constitutes all but a minute amount of its mass and one electron. Its volatility and small mass cause it to be labile, that is, comparatively easily driven from combination by other elements, and also to have a rather wide range of combination, associating itself with both noticeably negative and noticeably positive elements. The elements by which it may be displaced are determined by those with which it is associated. When in combination with negative elements, such as chlorine, oxygen and sulphur, it is displaced by positive elements; when in combination with carbon, as in a vast number of organic compounds, it is displaced by negative elements, such as chlorine and oxygen. The discovery that chlorine can readily substitute hydrogen in organic substances was, indeed, somewhat of a surprise to chemists in the period in which the Berzelian theories of electrochemical relations were in high favor, but the fact is not now so puzzling. As the knowledge of the composition and properties of acids increased, oxygen ceased to be regarded as deserving of the claim on which its name is based, and hydrogen came to be regarded as the acidifying principle. The introduction of volumetric methods and the discovery of indicators more sensitive and precise than the vegetable colors that were at first used, brought about a more liberal interpretation of the phenomena of acidity, alkalinity and neutrality. It was recognized that the acid condition depends largely on the position of the hydrogen in the molecule, as noted above. In further development of correct ideas on these points, the theory of ionization came into play.

As with all developments in science, it is difficult to find the point of origin of this theory. Wilhelm Pfeffer, a botanist, studied the phenomena of diffusion of organic liquids, such as sugar

solutions, through membranes to elucidate some of the laws of vegetable physiology; Dutrochet carried the investigations further and introduced the terms "endosmose" and "exosmose," and Graham made a long series of experiments elucidating some of the laws of these transudations, and also discovered that differences of physical conditions, which he distinguished by the terms "crystalloid" and "colloid" determined marked differences in the power to pass through membranes. He invented, in fact, a new method of separation of compounds, which he termed "dialysis," which has found extensive application in physical chemistry. The studies of the phenomena of solution that are included in the labors of Pfeffer, Dutrochet and Graham were made almost entirely with water as the solvent, and inasmuch as this is the liquid which functions in all normal biologic processes, the data derived from such studies have direct reference to such processes. It will not be necessary, therefore, to consider the effects of any other solvent. When any solid substance is dissolved in water, the properties both of solvent and solvend are modified. The solution has a higher boiling point and lower freezing point, and lower vapor pressure than pure water. It has also a strong tendency to flow through a membrane into water on the other side or even into water in direct contact. Water in contact with water of the same composition, of course, does not have any tendency to exchange, but if one portion contains, for instance, sugar, this solution at once tends to penetrate the mass of the pure water. This "urge" is termed "osmotic pressure." Osmotic pressure bears a definite relation to the amount of the substance in solution, and also to its molecular weight, and the same is true of the change in boiling point, freezing point and vapor pressure. The molecular weight of urea is 60; of cane sugar, 342. These weights, respectively, dissolved in equal volumes of pure water, will give the same osmotic pressure, and many other examples of this might be given. It was found, however, that many compounds, salts, strong acids and strong bases, give omotic pressures much higher than the rule just given requires. Sodium chloride has about twice the pressure; sulphuric acid about three times that which would be expected from the experiments with sugar and urea. The explanation of these exceptions is that by solution in the water, the acids, bases and salts have been modified so as to increase the number of active molecules, without producing

an actual decomposition. It is held that in the case of sodium chloride, for instance, a considerable number of the molecules have acquired electrical charges, distributed among the atoms, so that all the sodium atoms so affected are positively charged, and all the chlorine atoms negatively charged. In modern theories, the arrangement of the atoms of sodium chlorine when in crystalline form is not regarded as simple pairing, but is more complicated, but when in solution it may be, and generally is, assumed that each sodium atom has a chlorine atom as a companion. Except in very dilute solution, some of these molecules escape the assumption of electric charges, and the extent to which such assumption takes place affects very markedly the properties of the liquid. The assumption of electric charges is termed "ionization," the charged elements being termed "ions." The word is derived from a Greek word meaning "to wander or travel," because such charged elements pass to one or other of the poles of an electrical circuit. As only two electrical conditions exist, termed, respectively, positive and negative, indicated usually by the signs + and −, it follows that in compounds containing more than two elements, one at least of the ions must be compound. In hydrochloric acid, for example, all the hydrogen atoms may assume a positive charge and all the chlorine atoms a negative one, and if all the molecules in solution suffer such modification, the acid is said to be completely ionized. In nitric acid, however, there are three different elements and the distribution of these is $\overset{+}{H}, \overset{-}{NO_3}$. In ammonium nitrate we may have $\overset{+}{NH_4}, \overset{-}{NO_3}$.

The great practical, as well as theoretical, importance of these transformations is in reference to the action of acids and bases on living tissue. So far as their ordinary action upon non-living substances is concerned, ionization has comparatively little importance. It determines, it is true, the electric conductibility of liquids, whether produced by solution or by melting, for unless charged ions are present there is nothing to carry the current. Among the most important and frequent applications of acids are those for dissolving substances and neutralizing alkalies. In the latter application it has long been customary to employ solutions of equivalent—not equal—strength, termed standard solutions, the amount of acid present in a given volume being dependent on the molecular weight and the amount of hydrogen in the molecule *capable* of ionization, but not that which is actually

ionized, for the extent to which ionization takes place is determined by dilution and by other circumstances. Each acid has its special susceptibility to ionization under given conditions. It has already been pointed out that the acid property depends on the manner in which the hydrogen is associated in the molecule. In sulphuric acid, for instance, both hydrogen atoms are united with oxygen, and both may assume electrical charges and be displaced by other positive elements which will combine with the oxygen. In acetic acid, however, although there are four hydrogen atoms, only one is directly associated with oxygen, the others being solely in contact with carbon. Hence, acetic acid can yield but one positive ion for each molecule.

Another point of great importance is the susceptibility of each acid and alkali to suffer ionization at a given dilution. Great difference exists in this respect between the known acids and alkalies, and upon this depends largely the so-called *strength* of the substance. Sulphuric, nitric and hydrochloric acids are commonly termed "strong" acids; sodium hydroxide and potassium hydroxide are commonly termed "strong" alkalies, while acetic acid and ammonium hydroxide are said to be "weak." The distinction depends on the fact that at moderate dilution the strong acids and alkalies undergo extensive ionization so that a large proportion of their molecules are in a labile state and can act readily, while the weaker members of these groups are ionized to only a slight extent. Sulphuric, nitric and hydrochloric acid in moderate dilution in water have almost all their molecules in the ionized state, while acetic acid has but a small percentage in that condition. Similarly, potassium and sodium hydroxides in solution have a large proportion of the molecules ionized while ammonium hydroxide is affected to but a limited extent.

Of late years much investigation has been made of the relation of hydrogen-ion concentration to the growth of living organisms. In earlier studies in this field "total acidity" was mainly determined, but, as the statements made above show, this is not the factor that is most important. It has been found, for instance, that different species of some of the smaller animal organisms that live solely in water, select certain degrees of H-ion concentration. So it has been found that plants are influenced very materially by the H-ion concentration of the "soil solutions," that is, the water in the soil and the substances in solution therein. This

Epigæa repens, in nature. Acid soil. (Photograph by Clarence R. Shoemaker.)

Epigæa repens. Grown in pot with acid soil. (Grown and photographed by Frederick V. Coville, U. S. Department of Agriculture.)

phase of soil chemistry is comparatively new, and indications are that it may affect very seriously the existing theories of soil fertility and, indeed, relegate to the scrap heap a great deal of the data in agricultural chemistry that has been so assiduously gathered during the last fifty years or so. Soil analysis has been pursued very actively, and large amounts of money have been expended both in public and private laboratories. The U. S. Department of Agriculture has published many large volumes of maps and text, giving results of such analyses in various parts of the country. Except possibly pastoral life, agriculture is the oldest systematic human employment and for many centuries was conducted by empirical methods, mixed with not a little superstition. Isolated and imperfect experiments were made from time to time, in the centuries that followed immediately after the revival of learning, but the practical ignorance of the composition of the air and the lack of correct methods of soil analysis prevented valid conclusions. Van Helmont weighed and planted a small tree in a tub, having also weighed the earth therein. He watered the earth regularly, and after the tree had grown considerably, he weighed it and the earth, and finding a notable increase in the weight of the tree inferred that water had been converted into plant tissue. He, of course, overlooked the carbon taken up by decomposition of the carbon dioxide of the atmosphere.

A little before the middle of the last century, Justus Liebig presented before the British Association for the Advancement of Science, an extensive report on organic chemistry in its relation to agriculture and physiology, which appeared somewhat later in English as "Agricultural Chemistry." In this he discussed the assimilation of the forms of plant food, and developed very extensively the general principles on which the study of plant nutrition has been pursued. He disapproved of the view that the emission of carbon dioxide by plants in darkness is a process of respiration, attended by a corresponding absorption of oxygen, and his attitude on this matter prevented the acceptance of this view for many years. A long-established doctrine of agriculture is that the fundamental condition for nutrition of plants is a supply of carbon, nitrogen, potassium and phosphorus in such combinations as can be easily absorbed and diffused through the plant tissue. The carbon is derived principally from the carbon

dioxide of the air, the nitrogen can be satisfactorily obtained from ammonium compounds or nitrates, the potassium and phosphorus from the soluble compounds of these elements. Continued experimenting showed, however, that many accessory conditions are influential, and the list of ingredients necessary to make a soil fertile is being continually expanded. Bacteriology came into the question. The soil under ordinary conditions is very rich in this type of organisms and they play a much more important part than was formerly supposed. They transform both organic and mineral matter into forms readily assimilable by plants.

The data available at present concerning the relation of hydrogen-ion concentration is very extensive, even though the subject is comparatively new. The processes for determining the degree of concentration of the hydrogen-ion in a given liquid are of two types. For exact determination and fundamental research electrolytic methods are used, which involve a very elaborate form of apparatus and much care in conducting experiments. Fortunately, a series of synthetic colors has been prepared which, by marked change, indicate approximately all ordinary degrees of ionization of acids and alkalies. With solutions of such colors, experimenters may make close approximations to the ion concentration of animal secretions and excretions, soil solutions and fluids of vegetable structures. The colorimetric method has been extensively applied for study of wild flowers in their habitats. Among American investigators, Dr. Edgar T. Wherry, of the U. S. Bureau of Chemistry, has been very active and has accumulated a large amount of interesting and valuable data. The general method has been as follows: Small portions of soil close to the root of a given plant are mixed with pure water and after a short time has been allowed for the materials to mingle, a clear portion is tested on an ordinary spot-plate. The method is convenient and likely to be more accurate than transportation of the soil samples to the laboratory.

The investigations so far made along this line in different parts of the world show a fair degree of unanimity with a great number of plants and in very different regions, indicating that each species has its reaction preference, growing to best advantage in certain condition of ion concentration. Incidental points of value to agriculture are foreshadowed by some of the data obtained. Thus, it has been found that the potato can grow in a soil that has

Shortia galicifolia, growing in neutral soil in the garden of Doctor Wherry, Washington, D. C.

Shortia galicifolia, the same plant as in the upper plate, but after growing for one year in subacid soil. (Both illustrations from photographs by Doctor Wherry.)

a reaction inimical to one of its parasites, so that by maintaining the favorable reaction the potato will grow but not its injurious associate. It must, however, be borne in mind that under the operation of the law of natural selection, strains of this parasite may be developed more and more resistant to a certain reaction and thus, ultimately, the potato may not be so easily protected. This phase has already been noted in relation to the boll-weevil, which is essentially a subtropical insect, but since its extensive spread through the cotton belt, it has been noticed that, at the northern border, forms of the insect are developed which have a higher resistance to cold than the parent strain. Some plants show indifference to the soil conditions in this respect. Whether this is due to specific differences not evident in the morphology, cannot be said. We must not overlook the fact that the soil is not merely a storehouse of food which the plant absorbs, but a collection of organisms which are sensitive to the soil-solution. Extensive investigation shows that highly acid or alkaline soils or those charged heavily with neutral salts will interfere with many plants. The distinction between marine and fresh-water floras emphasizes this fact strongly. Farmers have long known that excess of acid phosphate will produce a "sour" soil and do injury. In general, bacterial organisms thrive best in a slightly alkaline medium, and most catalysts are also similarly affected. The fertility of limestone soil has been long known. Ordinarily, the acids of the soil are produced by the decomposition of leaf structure. Carbonic acid is one of the products of this decomposition and plays a great part in the changes that take place in soils.

The amount of hydrogen ionization in a given solution is expressed generally by a system of symbols, which at first seem somewhat complicated, but are regarded favorably by most workers in the field. The exact form of the symbol differs considerably in different countries, but American workers have generally agreed on the form pH. The starting-point of the concentration is 1 gram of hydrogen-ion in 1000 c.c. of water. Distinction must be noted between this and the conditions in an ordinary "normal" (N/1) solution. In the latter there is 1 gram *ionizable* hydrogen in 1000 c.c., but it is not all *ionized*. Starting with this hypothetical normal solution of ionized hydrogen, successive dilutions by ten are indicated by exponents attached to the symbol pH, but these exponents are the logarithms of the

quantities and not the direct expressions of the quantities themselves. The successive dilutions being the production of liquids containing less and less of the hydrogen-ion, the logarithms are minus, but the minus sign is not expressed, hence the somewhat puzzling fact that as the ion concentration diminishes the exponent increases. The following table shows the method of applying the symbol.

Amount of Ionized Hydrogen in 1000 c.c. of Water.	Proportion to Unit Standard.	Actual Logarithm.	Symbol as Written.
1 gram	1	0	pH_0
0.1 "	0.1	−1	pH_1
0.01 "	0.01	−2	pH_2
0.001 "	0.001	−3	pH_3
0.0001 "	0.0001	−4	pH_4
0.00001 "	0.00001	−5	pH_5
0.000001 "	0.000001	−6	pH_6
0.0000001 "	0.0000001	−7	pH_7

The last concentration is 1 gram of ionized hydrogen to 10,000,000,000 c.c. (ten billion c.c.). (This is the American and French method of notation of the higher numbers; in Great Britain a billion is a million millions.) This quantity of ionized hydrogen is that which occurs in pure water under the influence of its own ionizing power, so that pH_7 represents neutrality. The concentration of the hydrogen-ion cannot be brought below this proportion in any solution except by the counteraction of hydroxyl. In the pH_7 there is, of course, the status H, HO, the two ions being chemically equivalent, but representing, respectively, 1 and 17 parts by weight. To reduce the hydrogen-ion concentration below pH_7, hydroxyl may be added in the form of one of the soluble hydroxides, such as KHO, NaHO or NH_4HO. The extent to which the pH will be increased will depend upon the ionizability of the substance added. Sodium and potassium hydroxide will show stronger pH than ammonium hydroxide in equivalent amount, but it must again be noted that the *total alkalinity*, as determined by titration with acid, will be same 56 1 of KHO, 40 of NaHO and 35 of NH_4HO in the same volume of pure water will require the same amounts of acid to neutralize, but will not give the same ion concentration by either the electrical or colorimetric method.

Owing to the comparative awkwardness of the pH symbol, due originally to Sörensen, Wherry has suggested an entirely

different method of statement, beginning at neutrality and indicating the ion concentration by multiples for acidity and decimals for alkalinity. "Specific acidity," the term used in this direct method, is the amount of hydrogen-ion in 1000 c.c. of a given solution expressed in terms of the (approximate) ion concentration of pure water (1 gram in 10,000,000,000 c.c.). "Specific alkalinity" is the corresponding amount of equivalent hydroxyl-

pH	Specific Acidity.	pH	Specific Acidity.	Specific Alkalinity.
4.0	1000	7.0	1.0	1
.1	800	.1	0.8	1.25
.2	630	.2	0.63	1.6
.3	500	.3	0.5	2
.4	400	.4	0.4	2.5
.5	315	.5	0.32	3.15
.6	250	.6	0.25	4
.7	200	.7	0.2	5
.8	160	.8	0.16	6.3
.9	125	.9	0.13	8
5.0	100	8.0	0.10	10
.1	80	.1	0.08	12.5
.2	63	.2	0.063	16
.3	50	.3	0.05	20
.4	40	.4	0.04	25
.5	31.5	.5	0.032	31.5
.6	25	.6	0.025	40
.7	20	.7	0.02	50
.8	16	.8	0.016	63
.9	12.5	.9	0.013	80
6.0	10	9.0	0.010	100
.1	8	.1	0.008	125
.2	6.3	.2	0.0063	160
.3	5	.3	0.005	200
.4	4	.4	0.004	250
.5	3.15	.5	0.0032	315
.6	2.5	.6	0.0025	400
.7	2	.7	0.002	500
.8	1.6	.8	0.0016	630
.9	1.25	.9	0.0013	800
7.0	1	10.0	0.0010	1000

ion. As it is often desirable to make comparisons between the two systems, or conversions of one into the other, Wherry has prepared a table which is herewith transcribed from *Ecology* (1923, **3**, 346). The original suggestion appeared in *Jour. Wash. Acad. Sci.* (1921, **11**, 197).

Doctor Wherry has favored me with a number of illustrations of plants growing in special conditions of hydrogen-ion concentration, the conditions having been ascertained by the colorimetric methods above mentioned. Among the interesting data thus

obtained is the fact that the familiar rhododendron will thrive only in a distinctly acid soil. Already soil-analysis by this method has resulted in securing valuable data as to the adaptability of certain districts to certain kinds of plants.

Several forms of apparatus for the employment of the colorimetric methods have been devised. The electric methods are difficult and involve the use of expensive apparatus. A universal indicator of considerable range of sensibility was described by

Asplenium montanum. Acid soil. (Photograph by Clarence R. Shoemaker.)

F. H. Carr in the *Analyst* (1922, 47, 196). This is a mixture of several colors, and curiously, the color changes follow the spectrum order, the lowest pH to which the indicator is sensitive producing red and the highest violet. I have tried this indicator and found it in the main satisfactory. A difficulty is that methyl-red, which is one of the ingredients, is somewhat unstable and the colors produced in the intermediate pH values are apt to be less pronounced than desirable. The solution keeps pretty well, however, for several months and seems to be adapted to field work. For the use of individual colors, the "Double-wedge Comparator" made by Sterlin, of New Haven, Connecticut, is convenient,

and has been much used by Doctor Wherry, who has suggested some details in its construction.

It must be borne in mind that the reaction to colors or even the conductibility of a solution may not tell all about the effect of given ion concentration. The other substances in the liquid may take active part. It has recently been shown that while a 4 per cent. solution of acetic acid will corrode stainless steel, a malt vinegar of the same strength will not, and the same is true of a 4 per cent. pure citric acid solution as compared with the same strength of lemon-juice. The "buffer" effect of the other organic matters present is probably responsible for the difference.

In conclusion, I wish to express my obligations to Dr. Edgar T. Wherry for much assistance in the preparation of this paper, loan of slides, original photographs and literature on the subject.

The Earth-Current Measuring System at the Waterloo Magnetic Observatory. O. H. GISH. (*Terrestrial Mag.*, Sept., 1923.) —Although the Australian government has generously made available two strips of land adjacent to the observatory, each two rods wide and ten miles long, only about one-fifth of this grant is utilized. From the observatory the experimental lines extend two miles to the east and the same distance to the north. The observing station is connected to a point a mile to the east and to one a mile to the north, both by underground and by aerial lines, while only aerial lines join it to the points two miles away. "The greatest obstacle in the way of quantitative measurements of earth currents is the spurious effects which arise at the electrodes. Thus two electrodes made from a conducting solid, even though every precaution be taken to make them physically and chemically identical, will, when placed in medium such as water or soil, usually show differences of potential which are too great to be neglected in earth-current measurements." Of the metals tried, lead, iron, and perhaps cadmium, showed themselves superior for the purpose. The semi-aridity of the Waterloo region adds to the difficulties of proper electrode installation. At present only temporary electrodes, and these of lead, have been put in the earth. They are at such a depth below the surface as to be beyond the level of daily temperature changes. Each electrode is covered with a mound of earth.

For recording instrument a recording potentiometer is being tried out. This is Leeds and Northrup's multiple point recorder. The record is not continuous but is made at intervals. The observatory at Waterloo is just getting under way. Reports of the data obtained will henceforth appear from time to time. G. F. S.

FERROMAGNETISM AND ITS DEPENDENCE UPON CHEMICAL, THERMAL AND MECHANICAL CONDITIONS.*†

BY

L. W. McKEEHAN, Ph.D.

Research Laboratories of the American Telephone and Telegraph Company and the Western Electric Company, Incorporated.
Member of the Institute.

REVERSIBLE THERMAL EFFECTS.

The case of iron has received most attention and is the most complicated. As thoroughly annealed iron is heated from room temperature its I, H curve shrinks in dimensions and is so distorted that the initial slope and the maximum slope both increase while the value of I_∞ diminishes. This process continues until the Curie point is reached at about 770° C. and ferromagnetism disappears. At the Curie point the I, H curve may be thought of as consisting of two straight lines, an infinitesimal displacement from the origin along the I-axis, and an extension nearly parallel to the H-axis, the lack of exact parallelism being due to the paramagnetic susceptibility of the metal above its Curie point. The curves of Fig. 14, taken from the classic researches of Morris,[80] differ only in minor details from those more recently obtained at soft iron.[81] Fig. 15 gives from the same experiments the susceptibilities $(\mu - 1)$ for various field strengths and the maximum susceptibility, which occurs at lower fields as the temperature increases.

Paramagnetism above the Curie point is obviously closely connected with ferromagnetism below the same temperature. Both sets of phenomena have been extensively studied by Weiss and his pupils.[82, 83] An important review of temperature effects, with some new experimental work, has recently appeared,[84] and should be consulted for reference to most of the original papers.

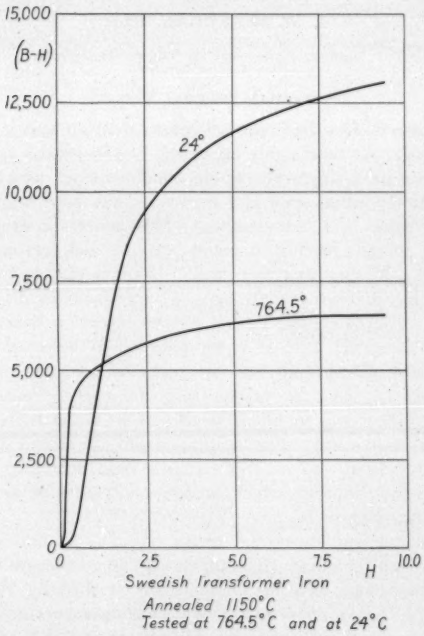
The behavior just described is typical of all ferromagnetic elements and alloys which are stable beyond the point where ferromagnetism disappears, and any deviation from this normal behavior may be regarded as indicating a change in structure, which

* Presented at a meeting held Thursday, February 28, 1924.

† Concluded from the May issue, p. 601.

will also be apparent in anomalous thermal expansion and in abnormal changes in other physical properties less closely connected with ferromagnetism. The Curie points, characteristic of the pure elements, are difficult to determine since they are much

FIG. 14.

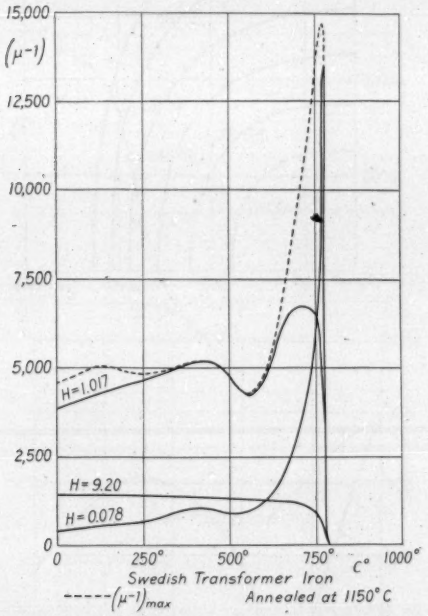


affected by small amounts of certain impurities, being generally lowered. That for cobalt may be taken at 1130° C. and that for nickel at 352° C. In all cases there seems to be a real difference between the Curie point for heating and for cooling, the former being the higher.

The cobalt-nickel series was examined in strong magnetic fields at various temperatures by Bloch,[31] whose results, replotted

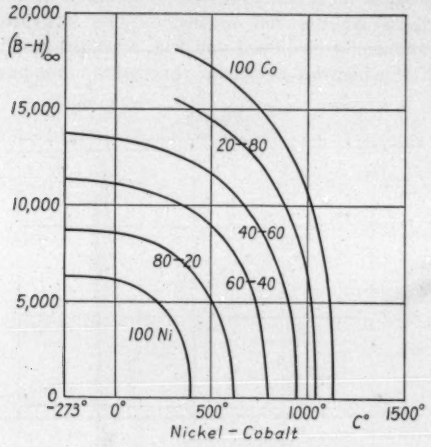
in Fig. 16, show no anomaly, the Curie points of the alloys lying between those of the end members. The nickel-iron series has been frequently examined and Fig. 17 illustrates the results obtained,[85, 33] which will be further referred to in the next section.

FIG. 15.



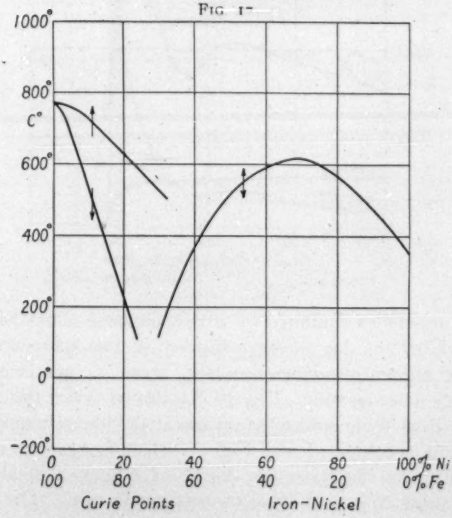
The anomalies exhibited by iron-manganese alloys have been explained [18, 53] as due to the formation of two compounds, that with the higher manganese content, about 14 per cent., being practically non-magnetic. Fig. 18 [18] is drawn to suit this explanation, the data being somewhat erratic at the lowest temperatures, and should be compared with Fig. 11, which agrees with the same view. Fig. 19 illustrates the degree of difference obtainable in various states of 12–13 per cent. manganese steel. The straight

FIG. 16.



Nickel - Cobalt

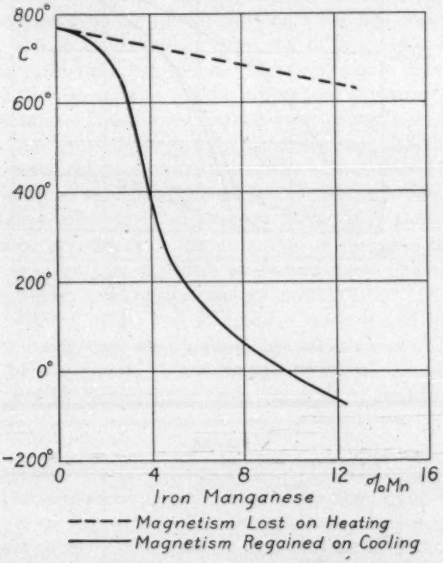
FIG. 1-



lines of Fig. 11 correspond to the more magnetic of the states shown in Fig. 19.

Silicon depresses the Curie point of iron about as shown in Fig. 20.[18] Comparing this with Fig. 9 there is no suggestion of a compound in this series as far as it extends. The brittleness

FIG. 18.



of ferro-silicon and the difference in crystal structures of the pure components [24, 86] suggest, however, that at higher silicon content anomalies like those produced by manganese are to be expected.

It has sometimes been proposed that all the alloys which are listed as non-magnetic merely have a very low Curie point. This is not a particularly helpful theory, for in some of the doubtful cases the Curie point, if it exists, is so near absolute zero that under any experimental conditions that can be maintained the substance would still be merely paramagnetic.[53] It is not even

FIG. 19.

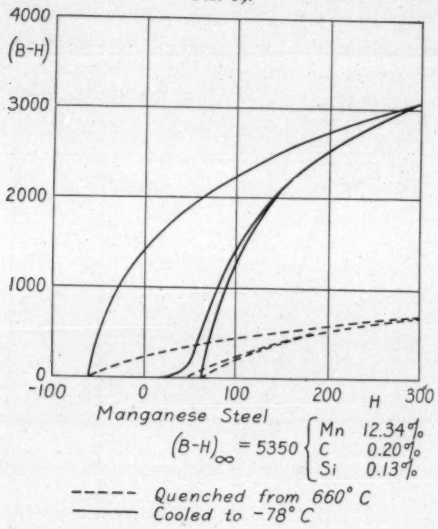
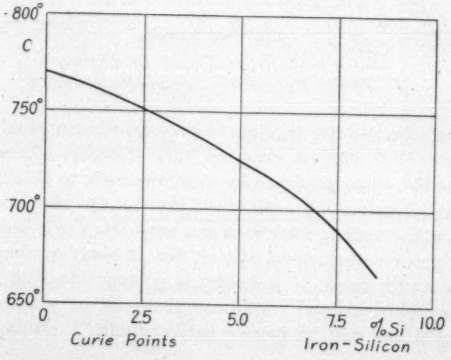

4000
(B-H)
3000
2000
1000
0
-100
0
100
200
H
300
Manganese Steel
(B-H)∞ = 5350
Mn 12.34%
C 0.20%
Si 0.13%
Quenched from 660° C
Cooled to -78° C

FIG. 20.


800°
C
750°
700°
650°
0
2.5
5.0
7.5
% Si
10.0
Curie Points
Iron-Silicon

possible to extrapolate to the Curie point by observations of paramagnetic susceptibility made wholly above it, the law of temperature variation of paramagnetism being particularly uncertain just where it would have to be relied upon.[82]

IRREVERSIBLE THERMAL EFFECTS.

What may be called the intrinsically irreversible thermal effects, the best example of which is near the low nickel end of the nickel-iron series have been supposed to be quite independent of chemical compound formation. This leaves only differences in crystal structure to account for the differences in magnetic properties in this case. This crystal change does, as we have seen, take place, but the great difference in hardness suggests that, as in chilled austenite the face-centred structure is maintained by strong forces tending to segregate two phases.[87] At such low temperatures ordinary methods of experiment do not give time for equilibrium to be arrived at. The structure of meteoric nickel-iron suggests, however, that there are really two stable phases between about 7 per cent. and 33 per cent. of nickel.[88, 89, 90, 43] On this view the cases of nickel and manganese steels would have much greater similarity than on the more orthodox view. The crystal structure of manganese is unknown,[91, 92] so we do not have a complete parallel between the cases as regards structure of the two phases.

STEEL.

Two peculiar features of the iron or iron-carbon temperature diagrams remain to be noted. The hump near 500° C. in the upper curves of Fig. 15 is as yet unexplained. It is suggested as possibly due to the Curie point of untransformed austenite, that is, of iron prevented from changing its space-lattice by the dissolved carbon. It has already been mentioned that at room temperature austenite must be slightly magnetizable so that it must change somewhere between 770° C. and room temperature. Immediately below the Curie point it should, like every other ferromagnetic substance, have its maximum susceptibility in rather low fields. The curves of Morris were obtained on a cooling sample and it is striking that on heating a well-annealed sample the maximum near 500° is hardly perceptible. Of course if residual austenite has all disappeared this is to be expected.

The other curious circumstance is the recurrence of a body-centred cubic structure near the melting point, which is, at the transition temperature,[84, 93] $A_4 = 1280°$ C., as at the previous transition in the opposite direction, $A_3 = 890°$ C., more paramagnetic than the face-centred cubic form. No explanation has yet been offered for this recurrence of a crystal form, but it strongly suggests the limited temperature range of complete miscibility in certain pairs of organic liquids. It is at least conceivable that perfectly pure iron would have no face-centred cubic structure at any temperature. This accords well with the known sensitiveness of the A_3 point to impurities.

A third peculiarity of the temperature susceptibility curves, occurring near 200° C., is known to be due to cementite which has its Curie point in this region.[51]

PERMANENT MAGNETS.

The effects of raising the temperature of permanent magnets are of two sorts, one being an accelerated aging or shaking down which produces partial but permanent demagnetization,[57] the other a temporary diminution of magnetic moment which is regained when the temperature again falls. The latter is much the more puzzling, suggesting as it does that magnetization can be built up against an opposing field (the self-demagnetizing field of the magnet) by simply cooling the magnet, that is, by a non-vector process. It is better probably to regard this reversible change as due to the change in spontaneous magnetization of the material postulated by Weiss,[94, 95] not for the moment attempting to decide why the new spontaneous magnetism happens to agree with that already present rather than with the direction of the magnetic field as calculated on the basis of a continuous medium. It is at least plausible that local fields agreeing in direction with the magnetization may be effective in directing elementary magnets, although the mean value of the field throughout any considerable volume has the contrary sign. The temperature coefficient for the reversible part depends upon the composition of the steel. Gumlich[18] gives $\alpha = -0.00050 + 0.00034\ p$ per degree C., in which p is the per cent. of carbon in steels quenched from 1100° C.

HEAT TREATMENTS.

High-temperature treatments are rarely completely separable from chemical treatments, even a vacuum being a chemical reagent in the sense that it will assist the escape of volatile impurities and disturb the equilibria of reactions having volatile phases. Most effects commonly attributed to heat treatment have accordingly been treated in preceding sections. The rates of chemical changes are so rapid at high temperatures that any process which is slow at 1000° C., or higher, is almost certainly being controlled by diffusion in the solid state, but this, as involving composition gradients, would be a chemical change in the larger sense here used. Besides the cases already discussed, attention is directed to Fig. 21,[18] showing the breakdown[96] of an aluminum alloy by overheating, and Fig. 22 showing (probably) slight oxidation of particularly pure iron. In the latter case the last anneal was followed by such slow cooling that the hardening is not attribu-

FIG. 21.

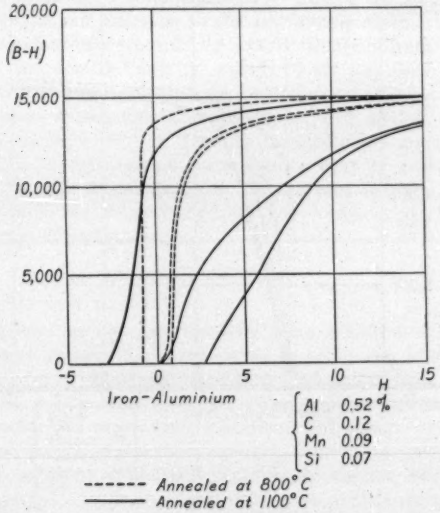
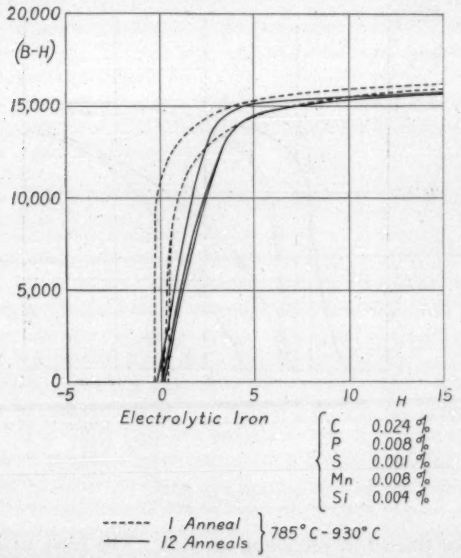


table to the occurrence of martensite. One other example may be referred to as representing the effects of cooling rate in rather high carbon steel.[97]

GRAIN-SIZE.

Changes in grain-size are sometimes reported to affect ferromagnetic properties.[98, 17] It is difficult to be sure in any case,

FIG. 22.



however, that the change in grain-size is not an index of the presence, absence, or state of aggregation of impurities which affect the magnetic behavior rather than the real cause of such differences as are observed. The grains are in any case so enormous in comparison with the atoms that it is hard to see how grain boundaries, *per se*, can introduce important changes in the condition of any appreciable number of atoms, at least in cases where, as in good magnetic iron, examination shows no appreciable transition layer between cleanly etched grains.

4. MECHANICAL EFFECTS.

There is only one numeric required to specify a temperature. A stress, on the other hand, even in homogeneous materials, requires at least three numerics and three orthogonal vectors for its complete specification at any point within the body of the sample and the stresses are in general different at different points. This added complexity is reflected in the greater complexity of the effects of stress upon magnetization, so that it will be necessary here to restrict our attention to the few simple cases which have been most extensively studied. In accordance with the plan of this paper, we will consider only the effects of stresses upon the *I, H* curve, leaving aside at this time all the reciprocal effects of magnetization upon dimensions commonly included under the term magnetostriction,[99, 100, 101, 102] as we omitted from consideration the effects of magnetization upon specific heat [103] and temperature.[83] We will, moreover, as in the previous section, deal first with temporary and reversible effects, later considering the effects of overstrain of various sorts.

SINGLE CRYSTALS.

Desirable as it would be to know the magnetic properties of single crystals, the data yet available are not sufficient to permit much generalization.[104] Experiments in which the same saturation values were obtainable in different directions are, it would seem, most worthy of confidence. The non-parallelism of field and magnetization vectors introduces into the general problem of crystal magnetization too many complications to permit as yet of any clear solutions. We will accordingly confine ourselves as yet to aggregates of crystals and must give preference to experimental work in which it is reasonable to suppose that the individual crystal axes have really been orientated at random. The conditions for such randomness are just beginning to be understood,[105, 106, 107, 108] so much early work must be reconsidered before too much is based upon it.

REVERSIBLE MECHANICAL EFFECTS. TENSION AND COMPRESSION.

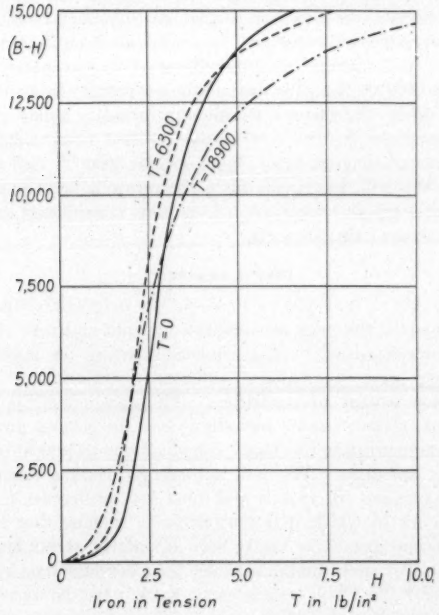
Since magnetization and stress are both vectors, the simplest case which presents itself is that in which the stress is entirely parallel to the magnetization. Of the two sorts of axial stress which are possible, tension is the easiest to apply in the long

samples which have been used in researches of this type. The effects in different materials are gratifyingly diverse.

IRON.

The nature of the changes produced in iron by tension has long been known. There is, in fact, little recent work in which

FIG. 23.



complete data are given. Fig. 23 [10] is fairly typical of the results usually obtained. Tension within the elastic limit increases the magnetization in a weak field and diminishes the magnetization in a strong field. The saturation value, I_∞, is little affected by elastic stresses. The normal magnetization curves for any two different tensions accordingly intersect each other at a second point besides

the origin. The intersection of curves drawn for adjacent values of the tension is, of course, a point on the envelope of the whole family of I, H curves. To each point of this envelope corresponds the tension, T, necessary to make the magnetization at the corresponding field strength a minimum. The value of T at the origin, which is the first point on the envelope, is zero, and T increases as we pass along the envelope in the direction of increasing H. Since experiments cannot, as we shall see, be carried beyond the elastic limit without introducing great changes in the magnetic material, it is clear that the envelope is limited in length.

Compression produces effects which are roughly opposite to those of tension but anomalies have been reported,[109] possibly to be ascribed to the difficulty of obtaining homogeneous longitudinal compressive stress except in prohibitively short rods.

Transverse effects, meaning by this term the effects upon magnetization in one direction produced by tensile or compressive stress in a direction at right angles, are much more difficult to measure and have been but little investigated. Samples of unusual shapes are necessary for such studies, so that self-demagnetization shall not be too great, and exact perpendicularity of the stress and the magnetic axis is essential if longitudinal effects are not to mask the results sought.[11] The conclusion generally drawn is that the transverse effect is opposite in sign to the longitudinal, but little is known of their relative magnitude in isotropic samples.

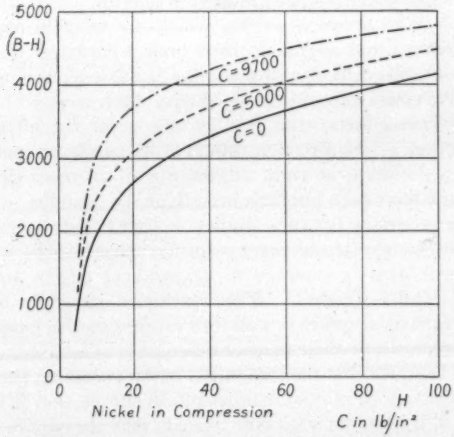
It will be noted that the statements in the preceding paragraphs apply exclusively to normal magnetization curves, and it has been assumed, although not explicitly stated, that the points of each such curve are obtained, after demagnetization in every case, for a sample under continued tension or compression. This is the usual procedure, for if the stress be altered while the applied field is maintained the effects are extremely erratic, probably because it is not possible, in ordinary materials at least, to make the variations in stress as well as its steady values homogeneous within the sample. Non-homogeneous stresses, particularly if rapidly changing, as in longitudinal vibrations, are known greatly to increase the magnetization due to weak fields, and are fairly efficient demagnetizers in zero applied field. The effect of vibrations, whether mechanically or magnetically maintained, upon the form of I, H curves, has been studied in some detail, but not as

yet in enough materials to make it possible to use the so-called "anhysteretic" curves for general comparisons.[10, 111-115, 22] Differences in the properties of materials are likely to be obscured by vibration, just as they are obscured by the use of overstrong fields in testing.

NICKEL.

In nickel the longitudinal effects of tension and compression are, as in cobalt, opposite to those in iron, except that no reversal

FIG. 24.



takes place in high fields. Williams[116] has given imperfect demagnetization as the cause for certain anomalous results reported in this connection. The degree in which the initial portions of the I, H curves of nickel can be altered by elastic stress is considerable, in fact, curves for nickel have been obtained[10] which differ more among themselves than do the initial portions of curves for cobalt and nickel in similarly soft conditions. Fig. 24[10] shows a less extreme case of compression effects in nickel.

COBALT.

What little has been done with cobalt indicates that the longitudinal effects of tension and compression in this metal are nearly opposite in character to those in iron. Compression favors and

tension hinders the magnetization by weak fields but the effects are both reversed in strong fields.[10]

TORSIONAL EFFECTS.

Torsional stresses are easier to apply than to measure and the results of torsion upon magnetization parallel to the axis of twist are, like those in crystals, and for the same reasons, very difficult to interpret. It was soon recognized that the extra complexity introduced by the use of non-homogeneous stress more than offset the experimental advantages so there is nothing of recent date to be included in our survey beside the review by Williams.[99] The difficulties have been reduced by the use of tubular samples, but here initial anisotropy is extreme, and if any remains at the time of test, it is particularly serious in cases like this, in which the transverse and longitudinal components of the various vectors involved cannot easily be interchanged.

IRREVERSIBLE MECHANICAL EFFECTS.

If any type of stress is carried beyond the elastic limit of the sample a new material is created, the magnetic properties suffering sudden changes of considerable amount, and isotropy being destroyed even if the dimensions of the piece are hardly altered. These effects are easily observed and may be used as proof of overstrain.[117, 118, 119, 120] If the deformation is carried far enough in one direction a fibrous structure develops, which it is very difficult to eradicate by subsequent heat-treatment.[121, 108] The nature of the changes in structure which occur depend upon the type of overstrain, drawing and rolling, for example, giving characteristically different results.[106, 108] This has undoubtedly been an unsuspected source of difficulty in many magnetic researches, especially where the small quantity of metal available, or other limitations, have forced the use of samples of unfavorable shapes and sizes. In attempting to normalize severely worked material, it is almost if not quite essential to raise the temperature high enough to blot out the existing crystal structure. Some ferromagnetic metals do not, unfortunately, permit this short of the melting point. Such ineradicable anisotropy as has been noted should furnish valuable clues to the effects of crystal orientation upon magnetization, upon which too few direct experiments have yet been performed. Fig. 25 and Table I present results of experiments[18, 122] upon a single lot of rolled iron sheets, showing

anisotropy due to rolling and its persistence after repeated aging processes at 100° C. The aging itself may be regarded as a reduction in favorable strains produced by cooling after a short anneal below the A_3 point. An additional example is given in

FIG. 25.

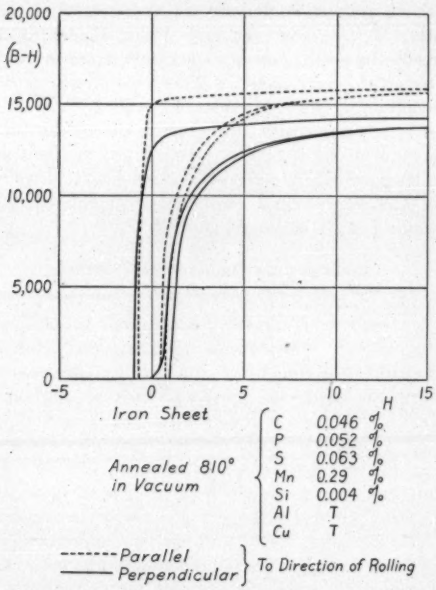
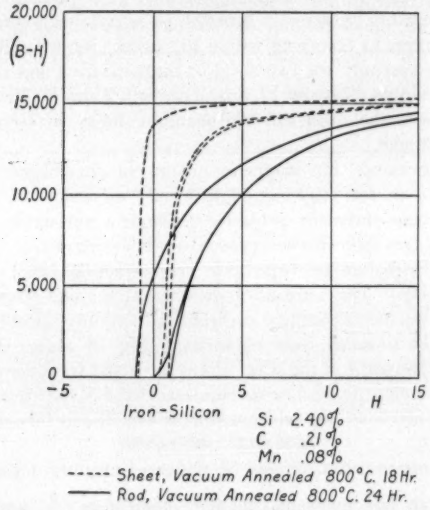


Fig. 26,[18] which shows differences due to the form of sample tested and the corresponding difference in mechanical history. Even extremely long annealing below 900° C. did not make the I, H curves similar.

5. THEORY.

Modern concepts of atomic structure and intra-atomic processes make no provision for gradual changes in energy of whatever kind. If we decide, therefore, as I think we must, that

FIG. 26.



the magnetization of a ferromagnetic metal involves changes in energy within its atoms, this part of the process at least had

TABLE I.

Effects of Grain Due to Rolling, and of Aging, in Magnetic Iron.
(After Gumlich.[18])

Analysis (per cent.)				
	C	0.046	Si	0.004
	P	0.052	Al	Trace
	S	0.063	Cu	Trace
	Mn	0.29		

*Coercivity.** (H_c)

Successive Heat Treatments.	Parallel to Direction of Rolling.	Perpendicular to Direction of Rolling.
Anneal at 810° C.	0.76	1.00
Age 600 hours at 100° C.	2.10	2.28
Anneal at 786° C.	0.775	1.01
Age 600 hours at 100° C.	2.18	2.38

*From $B_{max.} = 10{,}000$.

better be regarded as discontinuous, *i.e.*, as a quantum process. It will certainly be better, if it proves to be possible, to picture the entire process as occurring within the atoms, for we will thereby avoid the necessity for two sorts of magnetization and two sorts of constraints. The rest of this paper will contain a few speculations as to the form which a quantum theory of ferromagnetism must take.

The evidence for magnetic quanta in paramagnetic gases alluded to in the introduction is briefly as follows.[123, 124, 125] Silver atoms, thermally projected through a non-uniform magnetic field, are split into two groups, one being attracted, the other repelled, by the wedge-shaped pole near which the beam of silver vapor passes. There are no undeflected atoms and none having magnetic moments, positive or negative, certainly different from that of the so-called Bohr magneton. Fig. 27 shows diagrammatically the paths of the silver atoms projected from the oven, O. When no magnetic field exists between N and S the atoms arrive at P. When a strong field gradient is established they arrive at Q_1 and Q_2.

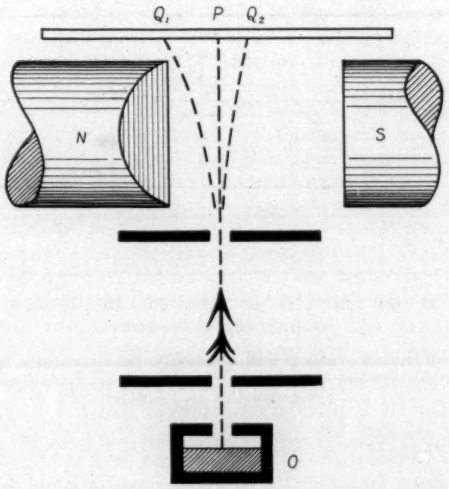
The interpretation offered by the experimenters is that every silver atom has the same angular momentum $\frac{h}{2\pi}$ and corresponding to this a magnetic moment $\frac{eh}{4\pi m}$, in which h is Planck's constant, and $\frac{e}{m}$ is the specific charge of the electron. The numerical value of this unit, the Bohr magneton, 9.2×10^{-21} e.m.u., is nearly five times the unit proposed by Weiss[126] on purely experimental bases. If the atoms have this magnetic moment the experiment shows further that in a magnetic field free silver atoms can only exist with their axes parallel to the field but with magnetic moments distributed nearly at random between the two possible directions.

A later interpretation[127] proposes that the angular momentum being still $\frac{h}{2\pi}$, the magnetic moment of each atom is not one, but two Bohr magnetons, and that the axes must be inclined at 60° to the magnetic field, so that the projection of the magnetic moment upon that direction, which is what the experiment measures, is plus or minus one magneton, as before. This explanation

is in somewhat better agreement with a rule discovered to hold in spectra, that angular momentum cannot change by more or less than one quantum $\frac{h}{2\pi}$ in any one event, such as the supposed possible transfer from one direction of magnetic axis to another.

On Bohr's theory of atomic structure [11, 12] an angular momentum $\frac{h}{2\pi}$ corresponds to an azimuthal quantum number

FIG. 27.

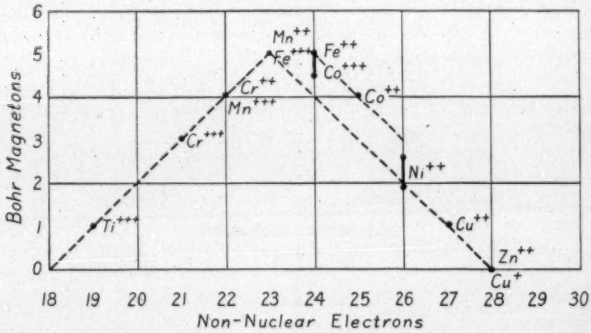


unity. In his notation this number is written as subscript to the total quantum number for any electron orbit, so that the angular momentum $\frac{h}{2\pi}$ is that for the orbits 1_1, 2_1, 3_1, etc. It is a striking fact that silver has only one such orbit, 5_1, not forming part of a completed group. The fact that only one of the forty-seven extra-nuclear electrons in the silver atom appears to be concerned in its magnetic behavior must mean that the angular momenta of the other forty-six electrons are so distributed that their resultant along any direction is zero at every instant. This implies a high

degree of symmetry, because the forty-six vectors to be disposed of have each an integral number of quanta of angular momentum.

We will now consider some evidence,[128] presented in Fig. 28, in regard to the magnetic structures of ions in aqueous solution, originally supposed to support a belief in the existence of the Weiss magneton. To the paramagnetic ions of the elements from atomic number 22 (titanium) to atomic number 28 (nickel), it is now proposed [129, 130] to ascribe from one to five Bohr magne-

FIG. 28.



tons. Still higher values will probably be necessary in dealing with the ions of the rare earths where paramagnetism is again conspicuous. It is necessary to suppose that five magnetons in Fe^{+++}, for example, are due to the composition of the magnetic moments of more than one orbit since no orbit is known to exist in this element for which the magnetic moment could be as great as this. The geometry of such composition of vectors is peculiar in that it permits only integral values for the projections of the component vectors upon the resultant axis. No detailed picture has yet been offered to show how this can be managed with the electron orbits supposed on Bohr's theory to be present in the steady state. It is worth speculating, however, in regard to probable arrangements in iron, cobalt and nickel, and the elements just preceding iron, a region, unfortunately, where spectroscopic evidence is not yet interpreted. We have, however, structures on either side of this gap to guide the interpolation. The arrange-

ments proposed by Bohr are based principally upon spectral evidence and should accordingly be applied to the solid state with great caution, but if they can be retained it will greatly simplify the problem.

In order to make a start we will have to decide upon the number of electron orbits we cannot affect by a magnetic field without destroying the crystalline structure, as we are sure we do not. Note in the first place that both calcium [131] and copper have face-centred cubic space-lattices and both are diamagnetic in the solid state. The diamagnetism of silver in the solid state indicates that in a face-centred cubic arrangement at least one electron per silver atom ceases to behave with respect to magnetic fields as it did in the gaseous state. The paramagnetism of aluminum which crystallizes in the same way as silver and has three more electrons per atom than the symmetrical and diamagnetic neon suggests that not more than two electron orbits per aluminum atom are made non-magnetizable in this crystalline arrangement. It should be noted, however, that the three valence electrons of aluminum are not assigned [12] to the same type of orbits so that it may be considered from one point of view to have but two valence electrons. Finally, the fact that solid argon has the same space-lattice [132] makes it probable that the electron orbit immobilized by the crystallization of silver is not itself necessary for the maintenance of this particularly symmetrical crystalline structure, but is merely trapped in its meshes, so that we must look deeper into the atom for the source of cohesion. It has already been suggested [35] that the M or three-quantum orbits rather than the N or four-quantum orbits are responsible for crystal structure in iron and nickel.

The usual picture of atomic structures will, in what follows, be strictly adhered to, and we will think of certain electrons as actually traversing orbits which project into the spaces between adjacent atoms. There is then a definite limit beyond which we cannot go without using up all the spaces available, for example, in the face-centred structure of argon. Each atom is surrounded by twelve others and the centres of these twelve adjacent atoms are at the corners of a rhombic dodecahedron. The normals to the faces of this figure from its centre are the directions in which the 4_1 electron orbits may be supposed to recede furthest from the nucleus without interfering with the adjacent atoms, and we will assume that, if possible, these orbits actually arrange themselves

so as to project in these directions. The fourteen faces are of two sorts, eight being isosceles triangles and six being squares. The normals to the latter are parallel to the cubic axes, or to the edges of the elementary cube, the normals to the former are parallel to the cube diagonals. The square faces, permitting as they do a symmetrical position of the orbital plane, and being of greater area, are first to be considered. The centres of these squares form a face-centred cubic space-lattice like that on which the atom centres are set, but so placed that both together form a simple cubic space-lattice of half the parameter.

One, two, three, four or six valence electrons could be accommodated by each of these meshes in the original lattice without any cumulative dissymmetry being thereby introduced into the arrangement as a whole. It is easy to account for one, and this requires no phase adjustment between the orbital motions of the single external electrons of neighboring atoms, no two external electron orbits being supposed to penetrate the same mesh. Two will be accommodated if the two electrons which must enter the same mesh do so alternately and from opposite directions. Six will be manageable in the same way if all six atoms which lie nearest along the cubic axes through the centre of each mesh each furnish one electron in turn, the time and space symmetry [133] of the model being thus completed. Three and four form parts of the completely symmetrical case and require no special consideration.

The body-centred cubic space-lattice can be treated in a similar way. Here each atom has eight neighbors and six adjacent meshes, all equal squares. There are, therefore, just the same possibilities of immobilizing external electron orbits as in the former case. The greater the number of electrons to be accommodated the more extensive must be the synchronism of their motions and the less stable the proposed structures will become. This may be why no element with a principal valence higher than four has a cubic space-lattice.[134]

The purpose of the preceding paragraphs is to show that, although it is possible for the atoms of a solid to have different electron distributions from the same atoms in a less orderly state, it is not often necessary. The fact that the elements with the greatest numbers of valence electrons have also the least sym-

metrical crystal structures suggests that the normal distributions are rather persistent than otherwise. We can now consider what internal structures can be assigned to the ferromagnetic elements and their immediate neighbors.

Proceeding backward through the series of atomic numbers, nickel, cobalt, and iron all possess cubic space-lattices, cobalt in addition having an hexagonal alternative the conditions for the occurrence of which are not well known.[26, 27] We can, therefore, take care of the 4_1 electrons, probably two in number for each, without any difficulty. Manganese has a space-lattice of low symmetry indicating, as does its high maximum valence, a low degree of atomic symmetry. Chromium and vanadium have cubic space-lattices and high maximum valence, and so are probably intermediate in structure between iron and manganese. Titanium, again, is probably less symmetrical internally, having an hexagonal structure. The crystal structure of scandium is unknown.

Since we have concluded that the orbits of the two outer electrons probably possessed by all these elements will, even in cubic arrangements, be prevented from exhibiting magnetic properties, it will be natural to assume that in less symmetrical arrangements they are at least as hard to orientate and can contribute nothing to ferromagnetism, or even to paramagnetism. We must look for the source of conspicuous magnetic properties, then, in the possibility of changing the orientations of one or more of the M orbits so as to make the vector sum of the magnetic moments of these orbits greater, or more nearly parallel to the impressed field, or both. Table II has been constructed so as to preserve in each of the ferromagnetic atoms a cubically symmetrical structure of elliptical M orbits to preserve crystalline symmetry, in which the excess circular M orbits may be supposed to orientate themselves freely under the influence of the magnetic field. The magnetizable part of the atom consists in this view of these excess 3_3 orbits, the 4_1 orbits being locked in the crystal structure as already suggested. The paramagnetic elements in the series differ from the ferromagnetic only in the incompleteness of the 3_1 and 3_2 levels, and the correspondingly greater constraint upon the possible orientations of the 3_3 orbits. The correctness of the proposed structures cannot be directly tested by comparison with published data but possible means of examination already exist.[135]

TABLE II.

Suggested Atomic Structures (in the Solid State).

Atomic Number.	Element.	Symbol.	Number of Electrons in Each Energy Level.			
			K	*L*	*M*	*N*
Z*			1_1	2_1 2_2	3_1 3_2 3_3	4_1 4_2 4_3 4_4
18	Argon	A	2	4 4	4 4 –	– – – –
19	Potassium	K	2	4 4	4 4 –	1 – – –
20	Calcium	Ca	2	4 4	4 4 –	2 – – –
21	Scandium	Sc	2	4 4	4 4 1	2 – – –
22	Titanium	Ti	2	4 4	4 4 2	2 – – –
23	Vanadium	V	2	4 4	4 4 3	2 – – –
24	Chromium	Cr	2	4 4	4 4 4	2 – – –
25	Manganese	Mn	2	4 4	4 4 5	2 – – –
26	Iron	Fe	2	4 4	6 6 2	2 – – –
27	Cobalt	Co	2	4 4	6 6 3	2 – – –
28	Nickel	Ni	2	4 4	6 6 4	2 – – –
29	Copper	Cu	2	4 4	6 6 6	1 – – –
30	Zinc	Zn	2	4 4	6 6 6	2 – – –

*Structures for Z = 18, 19, 20, 21, 22, 29, 30 according to Bohr.[12]

COMPARISON WITH EXPERIMENT.

Let us now examine very briefly some of the suggestive correspondences between the phenomena described in the earlier sections of this paper and the model here proposed for the ferromagnetic elements and for those which immediately precede them.

The first difficulty is that the saturation values in iron, cobalt and nickel form a descending series whereas the number of 3_3 orbits has been chosen to form an ascending series. It has already been suggested that the saturation value for cobalt may be considerably higher than reported work has yet indicated. In the case of nickel it might be supposed that four circular 3_3 orbits cannot be so set as to give as great a resultant as two or three can without so distorting the atom that one or both of the 4_1 orbits become unstable and are replaced by 3_3 orbits, thus reducing the disposable magnetic moment of the structure. A point which was not stressed, *i.e.*, the difficulty of attaining saturation even in very soft iron, well shown in Fig. 5, may be ascribed to a similar increase in 3_3 orbits in powerful fields, accompanied in this case, unlike that of nickel, by an increase in the magnetization. The reversal of magnetostriction in iron at high magnetizations[99] favors the same explanation. The 3_3 orbits cannot under the most favorable circumstances have their axes all in coincidence, to judge by the observed saturation values per atom.[130]

The second stumbling-block is the existence of hysteresis, the changes in energy corresponding to reorientations of orbits in a moderate magnetic field being extremely small and the corresponding radiation quanta being therefore insufficient to account for the energy lost from the conservative part of the system in an ordinary hysteresis loop. This difficulty disappears, however, in explaining another class of phenomena, the effects of stress. It must be assumed that in completely demagnetized ferromagnetic materials the symmetry of the atomic structures is not as high as is suggested by the crystal symmetry, but that the latter is merely an average symmetry due to the random arrangement of the 3_3 orbits from atom to atom. If it be attempted to make this arrangement symmetrical with respect to a given axis, the direction of the applied field, the dissymmetry exhibits itself in magnetostriction and in accordance with well-known principles the artificial production of similar changes in dimensions, by applied stress, increases the ease of magnetization. Now the atomic dissymetry, just assumed, must introduce greater differences between the energies of differently orientated orbits than are contemplated in the simple theory of an elementary magneton in a uniform field, and therefore greater dissipation of energy in any magnetizing process. It is significant that the application of the proper axial stress not only favors magnetization by freeing the magnetizable orbits from this influence, but at the same time greatly reduces hysteresis loss.[10] It may not be too much to hope that if an appropriate axial anisotropy could be introduced into any material that its hysteresis losses would fall to zero. The low hysteresis and easy magnetization of permalloy may be traceable to the compensating effects of two sorts of atomic dissymmetry, the initial magnetostrictions in nickel and iron being opposite, so that each tends when magnetized to apply stresses of the right sort upon the other. A similar explanation in higher fields would apply to the properties of iron-cobalt alloys with low cobalt content. The measurement of magnetostriction in alloys should throw further light on these questions.[136]

All the differences between magnetization curves of the same material under different conditions are naturally to be regarded, on this basis, as due to differences in the residual stresses and to

differences in crystal orientations which alter the type of distortion experienced by each atom in the crystal. When, however, different saturation values are reached, it is impossible to escape the conclusion that the number of magnetizable orbits has been altered, as it certainly is in chemical compounds of the ferromagnetic elements, the electrons in the 3_3 orbits being among the valence electrons in this region of atomic numbers.

The Heusler alloys contain, on this view, a chemical compound of the constituent metals in which the manganese is sufficiently ionized so as not to possess as many as five electron orbits of any one type, and in which in addition the manganese atom is so symmetrically surrounded that its remaining magnetizable orbits are nearly free to assume quantized orientations with respect to the applied field. The fact that one of the constituents may be varied and the fact that the magnetic alloy has a cubic space-lattice [78] both support this view.

The effects of rising temperature are simply to be regarded as due to the increasing energy of the atoms. When the amplitude of atomic vibration reaches such a value that the electron orbits of neighboring atoms penetrate both atomic structures the dissymmetry of each atom occasionally vanishes and susceptibility increases to a maximum. At the same time, however, the reduction in the constraint of the outer electron orbits deprives them of any opportunity to exchange angular momentum with the core, the specific heat falls [103] and ferromagnetism is replaced by paramagnetism. As might be expected, this is only the final stage of a process which begins to be appreciable at a much lower temperature, the Curie point being on this view a statistical end-point like the so-called range of the alpha-particle. A similar view of the Curie point as the last stage in a process beginning at a much lower temperature has long been upheld by Honda.[137]

It is interesting to note that in nickel, the susceptibility of which is increased by tension, a change in the same direction is effected by introducing transverse joints [138] which have so detrimental an effect in iron. It has been found also [139, 140] that magnetite shows effects under tension and compression analogous to those in nickel, showing that the sign of these effects is due to the structure of those parts of the iron atom which can be modified by chemical combination, *i.e.*, the N or M levels.

CONCLUSION.

Incomplete as our knowledge is, both of quantum processes and of magnetic processes, it is not too soon to point out that in the application of magnetic fields we have perhaps the only means for testing the quantum theory of atomic structure in the solid state, where, after all, some of the most interesting atoms usually stay. Neither is it too soon to attempt to coördinate our meagre information on these topics. If this contribution to the theoretical side of ferromagnetism suggests ways of improving magnetic materials, it will have served a useful purpose, however inexact in details it may prove to be. If the studies in ferromagnetism to which it may lead teach us more about the way in which the universe really acts, this result, intangible as it is, would be a sufficient justification for its presentation.

REFERENCES.

[1] SMITH, ARTHUR W., and HAMMOND, H. E.: *Phys. Rev.*, (2), **15**, 249–255 (1920).

[2] WÜRSCHMIDT, J.: *ZS. f. Phys.*, **9**, 379–394 (1922).

[3] LORENZ, H.: *ZS. f. techn. Phys.*, **2**, 63–71 (1921).

[4] HONDA, K.: *Dictionary of Applied Physics*. **5**, 516–526 (1922).

[5] WILLIAMS, E. H.: *Phys. Rev.*, (2) **12**, 158–166 (1918).

[6] WILLIAMS, E. H.: *Phys. Rev.*, (2) **14**, 348–351 (1919).

[7] WOLTJER, H. R.: *Amsterdam Proc.*, **26**, 613–625 (1923).

[8] WOLTJER, H. R., and KAMERLINGH ONNES, H. A.: *Amsterdam Proc.*, **26**, 626–634 (1923).

[9] RICHTMYER, F. K., and WARBURTON, F. W.: *Phys. Rev.*, (2) **21**, 721 (1923).

[10] EWING, J. A.: "Magnetic Induction in Iron and Other Metals," 3rd ed., revised (1900).

[11] BOHR, N.: *Ann. d. Phys.*, (4) **71**, 228–288 (1923).

[12] BOHR, N.: *Nature*, **112**, 29–44 (1923).

[13] SOMMERFELD, A.: "Atombau und Spektrallinien," 3rd ed. (1922).

[14] FOOTE, P. D., and MOHLER, F. L.: "The Origin of Spectra" (1922).

[15] ANDRADE, E. N. DA C.: "The Structure of the Atom" (1923).

[16] GUMLICH, E., and STEINHAUS, W.: *Elektrot. ZS.*, **36**, 675–677, 691–697 (1915).

[17] YENSEN, T. D.: *Amer. Inst. Elect. Eng.*, preprint (Feb. 5, 1924).

[18] GUMLICH, E.: *Wiss. Abh. d. phys.-techn. Reichsanstalt*, **4**, 269–420 (1918).

[19] GUMLICH, E.: *Wiss. Abh. d. phys.-techn. Reichsanstalt*, **5**, 323–328 (1923).

[20] FORTRAT, R., and DEJEAN, P.: *Comptes Rendus*, **177**, 627–630 (1923).

[21] YENSEN, T. D.: *Proc. Amer. Inst. Elect. Eng.*, **35**, 2455–2495 (1915).

[22] WÜRSCHMIDT, J.: *ZS. f. Phys.*, **12**, 128–164 (1922).

[23] DROZ, A.: *Arch. des sci. phys. et natur.*, (4) **29**, 204–224 (1910).

[24] HULL, A. W.: *Phys. Rev.*, (2) **9**, 84–87 (1917).
[25] WESTGREN, A., and LINDH, A. E.: *ZS. f. phys. Chem.*, **98**, 181–210 (1921).
[26] HULL, A. W.: *Phys. Rev.*, (2) **14**, 540–541 (1919).
[27] HULL, A. W.: *Phys. Rev.*, (2) **17**, 571–588 (1921).
[28] WEISS, P.: *Trans. Faraday Soc.*, **8**, 149–156 (1912).
[29] WILLIAMS, E. H.: *Phys. Rev.*, (2) **6**, 404–409 (1915).
[30] ANDREWS, MARY R.: *Phys. Rev.*, (2) **18**, 245–254 (1921).
[31] BLOCH, O.: *Arch. des sci. phys. et natur.*, (4) **33**, 293–308 (1912).
[32] Circular No. 58, Bureau of Standards (1916).
[33] YENSEN, T. D.: *J. Amer. Inst. Elect. Eng.*, **39**, 396–405 (1920).
[34] WÜRSCHMIDT, J.: *Phys. ZS.*, **23**, 499–500 (1922).
[35] MCKEEHAN, L. W.: *Phys. Rev.*, (2) **21**, 402–407 (1923).
[36] HONDA, K., and TAKAGI, H.: *Trans. Faraday Soc.*, **15**, 54–61 (1920).
[37] GUILLAUME, C. E.: *Proc. Phys. Soc. London*, **32**, 374–405 (1920).
[38] CHEVENARD, P.: *Comptes Rendus*, **172**, 594–596 (1921).
[39] CHEVENARD, P.: *Comptes Rendus*, **172**, 1655–1657 (1921).
[40] PORTEVIN, A.: *Comptes Rendus*, **172**, 445–447 (1921).
[41] INGERSOLL, L. R., and others: *Phys. Rev.*, (2) **16**, 126–132 (1920).
[42] HANSON, D., and HANSON, HILDA E.: *J. Iron and Steel Inst.*, **102**, 39–64 (1920).
[43] HANSON, D., and FREEMAN, J. R.: *Engg.*, **115**, 667–670 (1923).
[44] HANSON, D., and FREEMAN, J. R.: *Nature*, **111**, 861–862 (1923).
[45] ZSCHIESCHE, K.: *ZS. f. Phys.*, **11**, 201–214 (1922).
[46] KAPITZA, P.: *Camb. Phil. Soc. Proc.*, **21**, 511–516 (1923).
[47] ARNOLD, H. D., and ELMEN, G. W.: JOUR. FRANK. INST., **195**, 621–632 (1923).
[48] BARRETT, W. F., BROWN, W., and HADFIELD, R. A.: *J. Inst. Elect. Eng.*, **31**, 674–729 (1902).
[49] CHENEY, W. L.: *Bull. Bur. Stds.*, **18**, 609–635 (1922). Sci. Paper No. 463.
[50] HONDA, K.: *Tôhoku Univ. Sci. Rep.*, **11**, 119–129 (1922).
[51] DEE, A. A.: *Proc. Roy. Soc. London*, **104A**, 316–321 (1923).
[52] HADFIELD, R. A.: *Trans. Faraday Soc.*, **8**, 94–97 (1912).
[53] KAMERLINGH ONNES, H. A., HADFIELD, R. A., and WOLTJER, H. R.: *Proc. Roy. Soc. London*, **99A**, 174–196 (1921).
[54] RAWDON, H. S., and SILLERS, F.: *Bull. Bur. Stds.*, **18**, 637–653 (1922). Sci. Paper No. 464.
[55] GUMLICH, E.: *Wiss. Abh. d. phys.-techn. Reichsanstalt*, **5**, 379–390 (1922).
[56] HONDA, K., and MURAKAMI, T.: *Tôhoku Univ. Sci. Rep.*, **6**, 235–294 (1917).
[57] HONDA, K., and SAITO, S.: *Phys. Rev.*, (2) **16**, 495–500 (1920).
[58] GUMLICH, E.: *ZS. f. Phys.*, **14**, 241–252 (1922).
[59] HUNTER, M. A., and BACON, J. W.: *Trans. Amer. Electrochem. Soc.*, **37**, 513–524 (1921).
[60] HONDA, K.: *Ann. d. Phys.*, (4) **32**, 1003–1026 (1910).
[61] VOGEL, R.: *ZS. f. anorg. Chem.*, **99**, 25–49 (1917).
[62] OVERBECK, K.: *Ann. d. Phys.*, (4) **46**, 677–695 (1915).
[63] FRIVOLD, O. E.: *Ann. d. Phys.*, (4) **68**, 356–368 (1922).
[64] AUWERS, O. VON: *Jahrb. d. Rad.*, **17**, 181–229 (1920).

[85] GUILLET, L.: *Rev. de métallurgie*, **17**, 494–500 (1920).
[86] JASSONEIX, B. DU: Original Communications, Eighth International Congress of Applied Chemistry, **2**, 165–170 (1912). *Chem. Abs.*, 318 (1913).
[87] HILL, B. V.: *Phys. Rev.*, **24**, 321–336 (1907).
[88] KEESOM, W. H.: *Phys. ZS.*, **15**, 723–729, 816 (1914).
[89] GANS, R., and FONSECA, A.: *Ann. d. Phys.* (4) **61**, 742–752 (1920).
[70] BAIN, E. C.: *Trans. Amer. Inst. Min. Eng.*, **68**, 625–641 (1923).
[71] BURROWS, C. W.: *Elect. World*, **78**, 115–116 (1921).
[72] VOGEL, R.: *ZS. f. anorg. Chem.*, **116**, 231–242 (1921).
[73] COULSON, J.: *Phys. Rev.*, (2) **20**, 51–58 (1922).
[74] KIDO, K.: *Töhoku Univ. Sci. Rep.*, **10**, 471–478 (1922).
[75] OXLEY, A. E.: *Proc. Roy. Soc. London*, **101A**, 264–279 (1922).
[76] AUWERS, O. VON: *Ann. d. Phys.*, (4) **63**, 867–899 (1920).
[77] HEUSLER, F.: *ZS. f. Phys.*, **10**, 403–404 (1922).
[78] YOUNG, J. F. T.: *Phil. Mag.*, (6) **46**, 291–305 (1923).
[79] WEDEKIND, E.: *Trans. Faraday Soc.*, **8**, 160–168 (1912).
[80] MORRIS, D. K.: *Phil. Mag.*, (5) **44**, 213–254 (1897); *Proc. Phys. Soc. London*, **15**, 134–177 (1897).
[81] WEISS, P., and RENGER, K.: *Archiv. f. Elektrot.*, **2**, 406–418 (1914).
[82] WEISS, P., and FOËX, G.: *Arch. des sci. phys. et natur.*, (4) **31**, 5–19, 89–117 (1911).
[83] WEISS, P.: *Jour. de phys.*, (6) **2**, 161–182 (1921).
[84] DEJEAN, P.: *Annales de phys.*, (9) **18**, 171–272 (1922).
[85] HEGG, F.: *Arch. des sci. phys. et natur.*, (4) **29**, 592–617; **30**, 15–45 (1910).
[86] HULL, A. W.: *Phys. Rev.* (2) **10**, 661–696 (1917).
[87] BAIN, E. C.: *Amer. Inst. Min. Eng.*, preprint 1299–S (Feb. 18, 1924).
[89] FRAENKEL, W., and TAMMANN, G.: *ZS. f. anorg. Chem.*, **60**, 416–435 (1908).
[90] BENEDICKS, C.: *Rev. de métallurgie*, **8**, 85–107 (1911).
[90] COLVER-GLAUERT, E., and HILPERT, S.: *Proc. Faraday Soc.*, **8**, 134–148 (1912).
[91] BAIN, E. C.: *Chem. and Met. Engg.*, **28**, 21–24 (1923).
[92] BECKER, K., and EBERT, F.: *ZS. f. Phys.*, **16**, 165–169 (1923).
[93] WESTGREN, A., and PHRAGMÈN, GÖSTA: *J. Iron and Steel Inst.*, **105**, 241–273 (1923).
[94] WEISS, P.: *Jour. de phys.*, (4) **6**, 661–690 (1907).
[95] TERRY, E. M.: *Bull. Nat. Res. Counc.*, **3** [3], 113–164 (1922).
[96] GUMLICH, E.: *Archiv. f. Elektrot.*, **9**, 153–166 (1920).
[97] NUSBAUM, C., and CHENEY, W. L.: *Bull. Bur. Stds.*, **17**, 65–78 (1921). Sci. Paper No. 408.
[98] RUDER, W. E.: *Trans. Amer. Inst. Min. Eng.*, **47**, 569–586 (1913).
[99] WILLIAMS, S. R.: *Bull. Nat. Res. Counc.*, **3** [3], 214–224 (1922).
[100] QUIMBY, S. L.: *Bull. Nat. Res. Counc.*, **3** [3] 225–234 (1922).
[101] MCCORKLE, P.: *Phys. Rev.*, (2) **22**, 271–278 (1923).
[102] HEAPS, C. W.: *Phys. Rev.*, (2) **22**, 486–501 (1922).
[103] WEISS, P., PICCARD, A., and CARRARD, A.: *Arch. des sci. phys. et natur.*, (4) **42**, 378–401 (1916); **43**, 22–52, 199–216 (1917).

[104] KUNZ, J.: *Bull. Nat. Res. Counc.*, **3** [3], 165–213 (1922).
[105] POLANYI, M.: *ZS. f. Phys.*, **17**, 42–53 (1923).
[106] POLYANI, M., and WEISSENBERG, K.: *ZS. f. techn. Phys.*, **4**, 199–208 (1923).
[107] BAIN, E. C., and JEFFRIES, Z.: *Chem. and Met. Engg.*, **25**, 775–777 (1921).
[108] JEFFRIES, Z.: *Amer. Inst. Min. Eng.*, "Annual Lecture of Institute of Metals Division" (Feb. 18, 1924).
[109] SMITH, C. M., and SHERMAN, G. W.: *Phys. Rev.*, (2) **4**, 267–273 (1914).
[110] JONES, W. M., and MALLAM, J. E.: *Phil. Mag.*, (6) **27**, 649–659 (1914).
[111] ASHWORTH, J. R.: *Phil. Mag.*, (6) **27**, 357–370 (1914).
[112] WRIGHT, W. B.: *Phys. Rev.*, (2) **11**, 161–169 (1918).
[113] GANS, R.: *Ann. d. Phys.*, (4) **61**, 379–395 (1920).
[114] GANS, R.: *Ann. d. Phys.*, (4) **64**, 621–624 (1921).
[115] WAGGONER, C. W., and MOLBY, F. A.: *Phys. Rev.*, (2) **17**, 427 (1921).
[116] WILLIAMS, S. R.: *Phys. Rev.*, (2) **4**, 288 (1914).
[117] FRAICHET, L.: *Rev. de métallurgie*, **20**, 32–45 (1923).
[118] FRAICHET, L.: *Rev. de métallurgie*, **20**, 549–559 (1923).
[119] DEFOSSE, A.: *Rev. de métallurgie*, **20**, 268–269 (1923).
[120] KRÜZNER, H.: *Archiv. f. Elektrot.*, **12**, 234–238 (1923).
[121] OBERHOFFER, P., and HEGER, A.: *Stahl u. Eisen.*, **43**, 1151–1155 (1923).
[122] GUMLICH, E., and VOLLHARDT, E.: *Elektrot. ZS.*, **29**, 903–907 (1908).
[123] GERLACH, W., and STERN, O.: *ZS. f. Phys.*, **8**, 110–111 (1921).
[124] GERLACH, W., and STERN, O.: *ZS. f. Phys.*, **9**, 349–352 (1922).
[125] GERLACH, W., and STERN, O.: *ZS. f. Phys.*, **9**, 353–355 (1922).
[126] WEISS, P.: *Arch. des sci. phys. et natur.*, (4) **31**, 401–438 (1911).
[127] LANDÉ, A.: *Phys. ZS.*, **24**, 441–444 (1923).
[128] CABRERA, B.: *Jour. de phys.*, (6) **3**, 443–460 (1922).
[129] EPSTEIN, P. S.: *Science*, **57**, 532–533 (1923).
[130] GERLACH, W.: *Phys. ZS.*, **24**, 275–277 (1923).
[131] HULL, A. W.: *Phys. Rev.*, (2) **17**, 42–44 (1921).
[132] SIMON, F., and SIMSON, C. V.: *Naturwissenschaft*, **11**, 1015 (1923).
[133] KOLKMEIJER, N. H.: *Amsterdam Proc.*, **23**, 1419–1441 (1922).
[134] WYCKOFF, R. W. G.: JOUR. FRANK. INST., **195**, 183–210, 349–365, 531–552 (1923).
[135] DAVISSON, C. J., and KUNSMAN, C. H.: *Phys. Rev.*, (2) **22**, 242–258 (1923).
[136] HONDA, K., and KIDO, K.: *Tôhoku Univ. Sci. Rep.*, **9**, 221–232 (1920).
[137] HONDA, K.: *Tôhoku Univ. Sci. Rep.*, **3**, 165–170 (1914).
[138] WILLIAMS, E. H.: *Phys. Rev.*, **33**, 60–64 (1911).
[139] WILSON, E.: *Proc. Roy. Soc. London*, **101A**, 445–452 (1922).
[140] WILSON, E.: *Proc. Roy. Soc. London*, **103A**, 185–189 (1923).

Ortho-cresol-tetrachlorophthalein.—E. L. ARNOLD, of Cornell University (*Jour. Am. Chem. Soc.*, 1924, **46**, 489–493), has synthesized this compound by condensation of ortho-cresol and tetrachlorophthalic acid in the presence of fuming, anhydrous stannic chloride. It is an indicator, being colorless in the presence of acids and purple in the presence of alkalies. The end-point lies between P_H 8.5 and P_H 9.0. J. S. H.

OSCILLATIONS AND RESONANCE IN SYSTEMS OF PARALLEL CONNECTED SYNCHRONOUS MACHINES.*†

BY

H. V. PUTMAN.

Electrical Department, Union College.

POWER PULSATIONS BETWEEN THREE SYNCHRONOUS MACHINES IN PARALLEL.

This problem is very much the same as the previous calculation for the case of two machines. There is, however, another simultaneous differential equation to be solved and the work is rather more complicated. The same symbols will be used, subscripts 3 referring to machine No. 3.

The equation of the torques acting on the rotor of machine No. 1 is:

$$T_{s_1}(\theta_1 - \varphi_1) + T_{d_1} p\,(\theta_1 - \varphi_1) + I p^2 \theta_1 = f_1(t) \qquad (1)$$

For machine No. 2, there is a similar equation:

$$T_{s_2}(\theta_2 - \varphi_2) + T_{d_2} p\,(\theta_2 - \varphi_2) + I_2 p^2 \theta_2 = f_2(t) \qquad (2)$$

Likewise for machine No. 3:

$$T_{s_3}(\theta_3 - \varphi_3) + T_{d_3} p\,(\theta_3 - \varphi_3) + I_3 p^2 \theta_3 = f_3(t) \qquad (3)$$

Substituting $\psi = (\theta - \varphi)$ in these equations they become:

$$T_{s_1}\psi_1 + T_{d_1} p\psi_1 + I_1 p^2(\psi + \varphi_1) = f_1(t) \qquad (1a)$$

$$T_{s_2}\psi_2 + T_{d_2} p\,\psi_2 + I_2 p^2(\psi_2 + \varphi_2) = f_2(t) \qquad (2a)$$

$$T_{s_3}\psi_3 + T_{d_3} p\,\psi_3 + I_3 p^2(\psi_3 + \varphi_3) = f_3(t) \qquad (3a)$$

In these equations, $\varphi_2 = \frac{P_1}{P_2}\varphi_1$ and $\varphi_3 = \frac{P_1}{P_3}\varphi_1$ so that the φ's really constitute but one variable. The fourth equation, obtained from power considerations, is:

$$(T_{s_1}\psi_1 + T_{d_1} p\psi_1) + \frac{P_1}{P_2}(T_{s_2}\psi_2 + T_{d_2} p\psi_2) + \frac{P_1}{P_3}(T_{s_3}\psi_3 + T_{d_3} p\psi_3) = 0 \qquad (4)$$

This last equation follows from the fact that the variation in the power output of the total system is zero.

---

* Communicated by Dr. Ernst J. Berg, Associate Editor of this JOURNAL.

† Concluded from the May issue, p. 621.

Putting $p = j\omega$ for the permanent condition these equations become:

$$\left[\frac{T_{s_1}}{I_1} - \omega^2 + j\frac{T_{d_1}}{I_1}\omega\right]\psi_1 - \omega^2\varphi_1 = \frac{1}{I_1}f_1(t) \tag{1b}$$

$$\left[\frac{T_{s_2}}{I_2} - \omega^2 + j\frac{T_{d_2}}{I_2}\omega\right]\psi_2 - \omega^2\frac{P_1}{P_2}\varphi_1 = \frac{1}{I_2}f_2(t) \tag{2b}$$

$$\left[\frac{T_{s_3}}{I_3} - \omega^2 + j\frac{T_{d_3}}{I_3}\omega\right]\psi_3 - \omega^2\frac{P_1}{P_3}\varphi_1 = \frac{1}{I_3}f_3(t) \tag{3b}$$

$$(T_{s_1} + j\omega T_{d_1})\psi_1 + \frac{P_1}{P_2}(T_{s_2} + j\omega T_{d_2})\psi_2 + \frac{P_1}{P_3}(T_{s_3} + j\omega T_{d_3})\psi_3 = 0 \tag{4a}$$

Let

$$A_n = \left[\frac{T_{sn}}{I_n} - \omega^2 + j\frac{T_{dn}}{I_n}\omega\right]$$

$$B_n = \frac{P_1}{P_n}(T_{sn} + j\omega T_{dn})$$

$$C_n = \frac{P_1}{P_n}\omega^2, \quad F_n = \frac{1}{I_n}f_n(t)$$

Making these substitutions the four simultaneous differential equations become:

$$A_1\psi_1 + 0\psi_2 + 0\psi_3 - C_1\varphi_1 = F_1 \tag{1c}$$

$$0\psi_1 + A_2\psi_2 + 0\psi_3 - C_2\varphi_1 = F_2 \tag{2c}$$

$$0\psi_1 + 0\psi_2 + A_3\psi_3 - C_3\varphi_1 = F_3 \tag{3c}$$

$$B_1\psi_1 + B_2\psi_2 + B_3\psi_3 + 0\varphi_1 = 0 \tag{4c}$$

Solving these equations for ψ_1,

$$\psi_1 = \frac{\begin{vmatrix} F_1 & 0 & 0 & -C_1 \\ F_2 & A_2 & 0 & -C_2 \\ F_3 & 0 & A_3 & -C_3 \\ 0 & B_2 & B_3 & 0 \end{vmatrix}}{\begin{vmatrix} A_1 & 0 & 0 & -C_1 \\ 0 & A_2 & 0 & -C_2 \\ 0 & 0 & A_3 & -C_3 \\ B_1 & B_2 & B_3 & 0 \end{vmatrix}} \tag{5}$$

This determinant represents the variation in the angular displacement of the rotor of machine No. 1. The expression for ψ_2 would be exactly equal to the expression for ψ_1 in which subscripts 1 and 2 are interchanged. This can easily be seen by writing the determinant for ψ_2 and interchanging two rows and two columns.

The algebraic reductions involved in the evaluation of the

determinant (5) are too long to be included but the results may be summarized as follows:

$$\begin{aligned}\text{Denominator} &= d + jd' \\ &= (N_1\omega^4 - N_2\omega^2 + N_3) + j(M_1\omega^5 - M_2\omega^3 + M_3\omega) \end{aligned} \tag{6}$$

where

$$\left.\begin{aligned} N_1 &= C_1 + C_2\left(\frac{P_1}{P_2}\right)^2 + C_3\left(\frac{P_1}{P_3}\right)^2, \quad M_1 = Td_1 + Td_2\left(\frac{P_1}{P_2}\right)^2 + Td_3\left(\frac{P_1}{P_3}\right)^2 \\ N_2 &= D_1 + D_2\left(\frac{P_1}{P_2}\right)^2 + D_3\left(\frac{P_1}{P_3}\right)^2, \quad M_2 = A_1 + A_2\left(\frac{P_1}{P_2}\right)^2 + A_3\left(\frac{P_1}{P_3}\right)^2 \\ N_3 &= E_1 + E_2\left(\frac{P_1}{P_2}\right)^2 + E_3\left(\frac{P_1}{P_3}\right)^2, \quad M_3 = B_1 + B_2\left(\frac{P_1}{P_2}\right)^2 + B_3\left(\frac{P_1}{P_3}\right)^2 \end{aligned}\right\} \tag{7}$$

In these expressions,

$$\left.\begin{aligned} C_1 &= Ts_1 + \frac{Td_1 Td_2}{I_2} + \frac{Td_1 Td_3}{I_3}, \quad E_1 = \frac{Ts_1 Ts_2 Ts_3}{I_2 I_3} \\ D_1 &= \frac{Ts_1 Ts_2}{I_2} + \frac{Ts_1 Ts_3}{I_3} + \frac{Ts_1 Td_2 Td_3 + Td_1 Ts_2 Td_3 + Td_1 Td_2 Ts_3}{I_2 I_3} \\ A_1 &= \frac{Ts_1 Td_2 + Td_1 Ts_2}{I_2} + \frac{Ts_1 Td_3 + Td_1 Ts_3}{I_3} + \frac{Td_1 Td_2 Td_3}{I_2 I_3} \\ B_1 &= \frac{Td_1 Ts_2 Ts_3 + Ts_1 Td_2 Ts_3 + Ts_1 Ts_2 Td_3}{I_2 I_3} \end{aligned}\right\} \tag{8}$$

C_2, D_2, A_2, B_2, may be obtained directly from C_1, D_1, A_1, B_1, by interchanging subscripts 1 and 2. Similarly C_3, D_3, A_3, B_3, may be obtained from C_1, D_1, A_1, B_1, by interchanging subscripts 1 and 3.

The numerator of the determinant is:

$$\left.\begin{aligned} &\left[(n_2 + jn'_2)\left(\frac{P_1}{P_2}\right)^2 + (n_3 + jn'_3)\left(\frac{P_1}{P_3}\right)^2\right]F_1 \\ &- \left[(n_2 + jn'_2)\frac{P_1}{P_2}\right]F_2 - \left[(n_3 + jn'_3)\frac{P_1}{P_3}\right]F_3 \end{aligned}\right\} \tag{9}$$

where

$$\left.\begin{aligned} n_2 &= M_3 - M_1\omega^2, \quad n_3 = M_7 - M_6\omega^2 \\ n'_2 &= N_3\omega - Td_2\omega^3, \quad n'_3 = N_7\omega - Td_3\omega^3 \end{aligned}\right\} \tag{10}$$

in which

$$\left.\begin{aligned} M_7 &= \frac{Ts_2 Ts_3}{I_2}, \quad M_5 = \frac{Ts_2 Ts_3}{I_3}, \quad M_6 = Ts_2 + \frac{Td_2 Td_3}{I_2}, \quad M_4 = Ts_2 + \frac{Td_2 Td_3}{I_3} \\ N_7 &= \frac{Td_2 Ts_3 + Td_3 Ts_2}{I_2}, \quad N_5 = \frac{Td_2 Ts_3 + Td_3 Ts_2}{I_3}. \end{aligned}\right\} \tag{11}$$

Also $$F_n = \frac{a_n + jb_n}{I_n}.$$

The determinant in equation (5), which represents the angular

displacement of the rotor of machine No. 1, may now be written in the form of three Fourier's series as follows:

Due to prime mover No. 1,

$$\psi_1 = \frac{\sqrt{g^2 + g'^2}\ \sqrt{a_{n_1}^2 + b_{n_1}^2}}{I_1\sqrt{d^2 + d'^2}} \times \sin\left(n\omega_1 t + \tan^{-1}\frac{b_{n_1}}{a_{n_1}} + \tan^{-1}\frac{g'}{g} - \tan^{-1}\frac{d'}{d}\right) \quad (12)$$

where

$$g = n_2\left(\frac{P_1}{P_2}\right)^2 + n_3\left(\frac{P_1}{P_3}\right)^2$$

and

$$g' = n'_2\left(\frac{P_1}{P_2}\right)^2 + n'_3\left(\frac{P_1}{P_3}\right)^2$$

Due to prime mover No. 2,

$$\psi_1 = \frac{-\frac{P_1}{P_2}\sqrt{n_2^2 + n'^2_2}\ \sqrt{a_{n_2}^2 + b_{n_2}^2}}{I_2\sqrt{d^2 + d'^2}} \times \sin\left(n\omega_2 t + \tan^{-1}\frac{b_{n_2}}{a_{n_2}} + \tan^{-1}\frac{n'_2}{n_2} - \tan^{-1}\frac{d'}{d}\right) \quad (13)$$

Due to prime mover No. 3,

$$\psi_1 = -\frac{P_1}{P_3}\frac{\sqrt{n_3^2 + n'^2_3}\ \sqrt{a_{n_3}^2 + b_{n_3}^2}}{I_3\sqrt{d^2 + d'^2}} \times \sin\left(n\omega_3 t + \tan^{-1}\frac{b_{n_3}}{a_{n_3}} + \tan^{-1}\frac{n'_3}{n_3} - \tan^{-1}\frac{d'}{d}\right) \quad (14)$$

The torque variation of machine No. 1, in percentage based on the average torque of reciprocating machine No. 1, is approximately

$$\frac{\psi_1 T_{s_1}}{a_{o_1}} 100.$$

This is also the percentage power pulsation based on the I.H.P. of reciprocating machine No. 1.

The three Fourier's series representing the percentage power pulsation are therefore:

Due to prime mover No. 1,

$$\frac{\psi_1 T_{s_1} 100}{a_{o_1}} = \frac{T_{s_1}\sqrt{g^2 + g'^2}\ \sqrt{A_{n_1}^2 + B_{n_1}^2}}{I_1\sqrt{d^2 + d'^2}} \times \sin\left(n\omega_1 t + \tan^{-1}\frac{B_{n_1}}{A_{n_1}} + \tan^{-1}\frac{g'}{g} - \tan^{-1}\frac{d'}{d}\right) \quad (15)$$

Due to prime mover No. 2,

$$\frac{\psi_1 T_{s_1} 100}{a_{o_1}} = -\frac{T_{s_1} P_1}{P_2} \frac{\sqrt{n_2^2 + n'^2_2}\ \sqrt{A^2_{n_2} + B^2_{n_2}}}{I_2 \sqrt{d^2 + d'^2}} \times \sin\left(n\omega_2 t + \tan^{-1}\frac{B_{n_2}}{A_{n_2}} + \tan^{-1}\frac{n'_2}{n_2} - \tan^{-1}\frac{d'}{d}\right) \tag{16}$$

Due to prime mover No. 3,

$$\frac{\psi_1 T_{s_1} 100}{a_{o_1}} = -\frac{T_{s_1} P_1}{P_3} \frac{\sqrt{n_3^2 + n'^2_3}\ \sqrt{A^2_{n_3} + B^2_{n_3}}}{I_3 \sqrt{d^2 + d'^2}} \times \sin\left(n\omega_3 t + \tan^{-1}\frac{B_{n_3}}{A_{n_3}} + \tan^{-1}\frac{n'_3}{n_3} - \tan^{-1}\frac{d'}{d}\right) \tag{17}$$

where

$$A_n = \frac{a_n}{a_{o_1}} 100 \qquad \text{and} \qquad B_n = \frac{b_n}{a_{o_1}} 100.$$

Let $Y_1 T_{s_1}$, $Y_2 T_{s_1}$, $Y_3 T_{s_1}$, be respectively the amplitudes of these Fourier's series. The maximum possible power pulsation in percentage of the k.v.a. rating of machine No. 1 is:

$$T_{s_1}(Y_1 + Y_2 + Y_3) \times \frac{\text{I. H. P.}}{\text{Rated k.v.a.}} \tag{18}$$

where I.H.P. and k.v.a. refer to machine No. 1.

Whether or not this maximum value of power pulsation will occur in actual operation depends upon the relative speeds of the machines. If none of the harmonics in one torque curve are multiples of any of the harmonics in either of the other two torque curves, then this maximum value of power pulsation will ocur. If, however, two of the machines are identical the maximum value will not occur unless the machines are synchronized with the cranks in exactly the right position. If the cranks are not in this position the power pulsation is determined by the amplitude of the sum of the three Fourier's series added at the proper phase relation, corresponding to the angles between the cranks.

As an illustrative example of the calculation of the power pulsation between three machines consider the following three engine-driven units:

Unit No. 1.

Engine.—6-cylinder, 4-cycle, single acting, 16½″ × 24″, 120° between cranks. Flywheel $WR^2 = 146{,}308$.

Generator.—36 pole—500 k.v.a.—400 k.w.—.8 p.f.—200 R.P.M.—2300 volt—3 phase—60 cycle. $P_o = 1370$, Rotor $WR^2 = 30{,}900$.

Unit No. 2.—Same as unit No. 1.

Unit No. 3.

Engine.—4-cylinder, 4-cycle, single acting, 180° between cranks. Flywheel $WR^2 = 1{,}681{,}000$.

Generator.—48 pole—625 k.v.a.—500 k.w.—.8 p.f.—150 R.P.M.—2300 volt—3 phase—60 cycle. Generator $WR^2 = 86{,}000$. $P_o = 1920$.

The manufacturer of the above engines wished to know how severe the hunting of the system would be with one cylinder of each engine missing fire and whether it would be possible to keep the machines in step under such conditions.

It was therefore necessary to construct torque curves for the above engines with one cylinder missing. In percentage, based on the average torque of the six-cylinder engine, the coefficients for the various harmonics of the Fourier's series with one cylinder missing are as follows:

6-cylinder.		1st.	2nd.	3d.	4th.
Sines	A_n	−44.17	12.86	50.28	−6.051
Cosines	B_n	11.08	54.63	1.042	−2.730
4-cylinder.					
Sines	A_n	−64.84	102.16	−54.34	−79.74
Cosines	B_n	71.30	−10.69	−57.83	81.21

The necessary calculations for obtaining the three Fourier's series (15), (16), and (17), are shown in Plate III. The maximum value of the power pulsation is seen to be 442 per cent. It is probably impossible for such a large value of power pulsation to occur for it is extremely likely that the machines would not stay in step under this condition. The reason for this extreme value of power pulsation can be seen after a study of the resonance frequencies in this combination of three machines.[8] It will be shown that the frequencies of the lowest impressed harmonics which correspond to half the revolutions come extremely close to the resonance frequencies.

A similar calculation for the power pulsation with all cylinders firing normally gave only 17.1 per cent., based on the k.v.a.

[8] These are discussed is the next section on "Resonance Frequencies in Synchronous Machines."

PLATE III.

Tabulation for Constants.

$T_{s_1}=8.674\times10^5$	$T_{d_1}=8.674\times10^3$	$I_1=5503$	$I_1\,I_3=3019.8\times10^5$	$(P_1/P_2)^2=1.00$
$T_{s_2}=8.674\times10^5$	$T_{d_2}=8.674\times10^3$	$I_2=5503$	$I_2\,I_3=3019.8\times10^5$	$(P_1/P_2)^2=.5625$
$T_{s_3}=21.61\times10^5$	$T_{d_3}=21.61\times10^3$	$I_3=54{,}876$	$P_1/P_2=1.00$	
$T_{s_1}\,T_{s_2}\,T_{s_3}=1625.9\times10^{15}$	$T_{d_1}\,T_{d_2}\,T_{d_3}=1625.9\times10^9$	$I_1\,I_3=302.8\times10^5$	$P_1/P_3=.75$	

$X_1=T_{d_1}\,T_{s_2}+T_{d_2}\,T_{s_1}=$ 150.48×10^5	$X_1\div I_2=$ 2.735×10^6	$X_1\div I_1=$ 2.735×10^6	$T_{d_1}\,T_{d_2}\,T_{d_3}\div I_1\,I_2=$ $.054\times10^6$	$T_{d_1}\,T_{d_2}=75.24\times10^6$
$X_2=T_{d_1}\,T_{s_3}+T_{d_3}\,T_{s_1}=$ 374.89×10^5	$X_2\div I_3=$ $.683\times10^6$	$T_{d_1}\,T_{d_2}\,T_{d_3}\div I_1\,I_3=$ $.005\times10^6$	$X_2\div I_1=$ 6.812×10^6	$T_{d_1}\,T_{d_3}=187.4\times10^6$
$X_3=T_{d_2}\,T_{s_3}+T_{d_3}\,T_{s_2}=$ 374.89×10^5	$T_{d_1}\,T_{d_2}\,T_{d_3}\div I_2\,I_3=$ $.005\times10^6$	$X_3\div I_3=$ $.683\times10^6$	$X_3\div I_2=$ 6.812×10^6	$T_{d_2}\,T_{d_3}=187.4\times10^6$
Adding columns	$A_1=3.423\times10^6$	$A_2=3.423\times10^6$	$A_3=13.678\times10^6$	$T_{s_1}\,T_{s_2}=75.24\times10^{10}$
				$T_{s_1}\,T_{s_3}=187.4\times10^{10}$
				$T_{s_2}\,T_{s_3}=187.4\times10^{10}$
				$y_1=T_{d_1}\,T_{s_2}\,T_{s_3}$ 1625.9×10^{13}
				$y_2=T_{d_2}\,T_{s_1}\,T_{s_3}$ 1625.9×10^{13}
				$y_3=T_{d_3}\,T_{s_1}\,T_{s_2}$ 1625.9×10^{13}
				$y_0=y_1+y_2+y_3$ 4877.7×10^{13}

$C_1=T_{s_1}+\dfrac{T_{d_1}\,T_{d_2}}{I_2}+\dfrac{T_{d_1}\,T_{d_3}}{I_3}=88.45\times10^4$	$E_1=\dfrac{T_{s_1}\,T_{s_2}\,T_{s_3}}{I_2\,I_3}=53.84\times10^8$	$B_1=y_0\div I_2\,I_3$ 1.615×10^8	
$C_2=T_{s_2}+\dfrac{T_{d_1}\,T_{d_2}}{I_1}+\dfrac{T_{d_2}\,T_{d_3}}{I_3}=88.45\times10^4$	$E_2=\dfrac{T_{s_1}\,T_{s_2}\,T_{s_3}}{I_1\,I_3}=53.84\times10^8$	$B_2=y_0\div I_1\,I_3$ 1.615×10^8	
$C_3=T_{s_3}+\dfrac{T_{d_2}\,T_{d_3}}{I_2}+\dfrac{T_{d_1}\,T_{d_3}}{I_1}=222.9\times10^4$	$E_3=\dfrac{T_{s_1}\,T_{s_2}\,T_{s_3}}{I_1\,I_2}=537.0\times10^8$	$B_3=y_0\div I_1\,I_2$ 16.11×10^8	

PLATE III—*Continued.*

$T_{s_1} T_{d_2} T_{d_3} = 1625.9 \times 10^{11}$

$T_{s_2} T_{d_1} T_{d_3} = 1625.9 \times 10^{11}$

$T_{s_3} T_{d_1} T_{d_2} = 1625.9 \times 10^{11}$

$Z = \Sigma = 4877.7 \times 10^{11}$

$$D_1 = \frac{T_{s_1} T_{s_2}}{I_2} + \frac{T_{s_1} T_{s_3}}{I_3} + \frac{Z}{I_2 I_3} = 1.725 \times 10^8$$

$$D_2 = \frac{T_{s_1} T_{s_2}}{I_1} + \frac{T_{s_2} T_{s_3}}{I_3} + \frac{Z}{I_1 I_3} = 1.725 \times 10^8$$

$$D_3 = \frac{T_{s_2} T_{s_3}}{I_2} + \frac{T_{s_1} T_{s_3}}{I_1} + \frac{Z}{I_1 I_2} = 6.971 \times 10^8$$

$$M_1 = T_{d_1} + T_{d_2}\left(\frac{P_1}{P_2}\right)^2 + T_{d_3}\left(\frac{P_1}{P_3}\right)^2 = 29.50 \times 10^5$$

$$M_2 = A_1 + A_2\left(\frac{P_1}{P_2}\right)^2 + A_3\left(\frac{P_1}{P_3}\right)^2 = 14.54 \times 10^8$$

$$M_3 = B_1 + B_2\left(\frac{P_1}{P_2}\right)^2 + B_3\left(\frac{P_1}{P_3}\right)^2 = 12.29 \times 10^9$$

$$M_4 = \left(T_{s_2} + \frac{T_{d_2} T_{d_3}}{I_2}\right) = 8.708 \times 10^5$$

$$M_5 = \frac{T_{s_2} T_{s_3}}{I_2} = .342 \times 10^8$$

$$M_6 = \left(T_{s_2} + \frac{T_{d_2} T_{d_3}}{I_2}\right) = 2.194 \times 10^6$$

$$M_7 = \frac{T_{s_2} T_{s_3}}{I_2} = 3.405 \times 10^8$$

$$N_1 = C_1 + C_2\left(\frac{P_1}{P_2}\right)^2 + C_3\left(\frac{P_1}{P_3}\right)^2 = 302.3 \times 10^4$$

$$N_2 = D_1 + D_2\left(\frac{P_1}{P_2}\right)^2 + D_3\left(\frac{P_1}{P_3}\right)^2 = 7.371 \times 10^8$$

$$N_3 = E_1 + E_2\left(\frac{P_1}{P_2}\right)^2 + E_3\left(\frac{P_1}{P_3}\right)^2 = 409.7 \times 10^8$$

$$N_4 = T_{d_2} = .8674 \times 10^4$$

$$N_5 = \left(\frac{T_{d_2} T_{s_2} + T_{d_3} T_{s_2}}{I_2}\right) = 68.2 \times 10^4$$

$$N_6 = T_{d_3} = 21.61 \times 10^3$$

$$N_7 = \left(\frac{T_{d_2} T_{s_3} + T_{d_3} T_{s_2}}{I_2}\right) = 681.2 \times 10^4$$

PLATE III—*Continued.*

Machine No. 1. $\omega_1 = 10.47$

n	1	2	3	4
$n\omega_1$	10.47	20.94	31.41	41.88
$n^2\omega_1^2$	109.6	438.5	986.6	1754
$n^3\omega_1^3$	1148	9183	309.9×10^2	734.6×10^2
$n^4\omega_1^4$	120.2×10^2	1923×10^2	9734×10^2	3076×10^3
$n^5\omega_1^5$	125.8×10^3	4026×10^3	3057×10^4	1288×10^5
$d'_n = +M_1n^5\omega_1^5 - M_2n^3\omega_1^3 + M_3n\omega_1$	-1.130×10^8	110.0×10^8	4898×10^8	$27{,}830\times10^8$
$d_{n_1} = N_1n^4\omega_1^4 - N_2n^2\omega_1^2 + N_3$	-34.80×10^8	2990×10^8	$22{,}563\times10^8$	$80{,}468\times10^8$
$n_2 = -M_4n^2\omega_1^2 + M_5$	$-.6124\times10^5$	-3.476×10^5	-8.249×10^5	-14.93×10^5
$n'_2 = -N_4n^3\omega_1^3 + N_5n\omega_1$	-2.817×10^6	-65.382×10^6	-247.45×10^6	-608.44×10^6
$n_3 = -M_6n^2\omega_1^2 + M_7$	1.00×10^5	-6.219×10^5	-18.25×10^5	-35.09×10^5
$n'_3 = -N_6n^3\omega_1^3 + N_7n\omega_1$	46.506×10^6	-55.826×10^6	-455.86×10^6	-1302.4×10^6
$y = \left(\frac{P_1}{P_2}\right)^2 n_2 + \left(\frac{P_1}{P_3}\right)^2 n_3$	-4.99×10^6	-697.4×10^6	-1852×10^6	-3467×10^6
$y^1 = \left(\frac{P_1}{P_2}\right)^2 n'_2 + \left(\frac{P_1}{P_3}\right)^2 n'_3$	23.34×10^6	-96.78×10^6	-503.5×10^6	-1341×10^6

Machine No. 2. $\omega_2 = 10.47$

$n\omega_2$	10.47	20.94	31.41	41.88
$n^2\omega_2^2$	109.6	438.5	986.6	1754
$n^3\omega_2^3$	1148	9183	309.9×10^2	734.6×10^2
$n^4\omega_2^4$	120.2×10^2	1923×10^2	9734×10^2	3076×10^3
$n^5\omega_2^5$	125.8×10^3	4026×10^3	3057×10^4	1288×10^5
$d'_{n_2} = +M_1n^5\omega_2^5 - M_2n^3\omega_2^3 + M_3n\omega_2$	-1.130×10^8	110.0×10^8	4898×10^8	$27{,}830\times10^8$
$d_{n_2} = N_1n^4\omega_2^4 - N_2n^2\omega_2^2 + N_3$	-34.80×10^8	2990×10^8	$22{,}563\times10^8$	$80{,}468\times10^8$
$n_2 = -M_4n^2\omega_2^2 + M_5$	$-.6124\times10^5$	-3.476×10^5	-8.249×10^5	-14.93×10^5
$n'_2 = -N_4n^3\omega_2^3 + N_5n\omega_2$	-2.817×10^6	-65.382×10^6	-247.45×10^6	-608.44×10^6

Machine No. 3. $\omega_3 = 7.85$

$n\omega_3$	7.85	15.70	23.55	31.40
$n^2\omega_3^2$	61.62	246.50	554.60	986.0
$n^3\omega_3^3$	483.7	3870	13.06×10^3	30.96×10^3
$n^4\omega_3^4$	3797	60.76×10^3	307.6×10^3	972.1×10^3
$n^5\omega_3^5$	29,809	953.9×10^3	7244×10^3	3052×10^4
$d'_{n_3} = +M_1n^5\omega_3^5 - M_2n^3\omega_3^3 + M_3n\omega_3$	34.94×10^8	-88.35×10^8	527.5×10^8	4888×10^8
$d_{n_3} = N_1n^4\omega_3^4 - N_2n^2\omega_3^2 + N_3$	70.28×10^8	429.5×10^8	5620×10^8	$22{,}528\times10^8$
$n_3 = -M_6n^2\omega_3^2 + M_7$	2.055×10^8	-2.005×10^8	-8.77×10^8	-18.25×10^8
$n'_3 = -N_6n^3\omega_3^3 + N_7n\omega_3$	4310×10^4	2330×10^4	$-12{,}200\times10^4$	$-45{,}500\times10^4$

PLATE III—*Continued.*

Machines No. 1.

Engine rating—6-cyl., 4-cycle, single acting, 16½″×24″. One cyl. missing.

Generator rating—36 p.—500 k.v.a.—400 k.w.—.8 p.f.—200 R.P.M.—2300 v.—3 φ—60~

$P_{o_1} = 1370$ | $\omega_1 = 10.47$	WR^2 of Gen. = 30,900
$T_1 = 5250$ H.P. ÷ R.P.M. = $.1407 \times 10^5$	WR^2 of Flywheel = 146,308
$T_{o_1} = T_1 P_{o_1} \times$ poles ÷ 2 k.w. = 8.674×10^5	Total $WR^2 = 177,208$
$T_{d_1} = 8.674 \times 10^5$	$I_1 = WR^2 \div 32.2 = 5503$

n	1	2	3	4
$A_{n_1} = 100\ (a_{n_1} \div a_{o_1})$ †	−44.17	12.86	50.28	−6.051
$B_{n_1} = 100\ (b_{n_1} \div a_{o_1})$ †	11.08	54.63	1.042	−2.730
$E_{n_1} = \sqrt{A_{n_1}^2 + B_{n_1}^2}$	45.60	56.15	50.28	6.635
$Q_{n_1} = E_{n_1} \div I_1$	$.8286 \times 10^{-2}$	1.020×10^{-2}	$.9137 \times 10^{-2}$	$.1206 \times 10^{-2}$
$d_{n_1} =$	-34.80×10^5	2990×10^5	$22{,}563 \times 10^5$	$80{,}468 \times 10^5$
$d'_{n_1} =$	1.130×10^5	110.0×10^5	4898×10^5	$27{,}830 \times 10^5$
$g_n =$	-4.99×10^6	-697.4×10^6	-1852×10^6	-3467×10^6
$g'_n =$	23.34×10^6	-96.78×10^6	-503.5×10^6	-1341×10^6
$H_{n_1} = \sqrt{g_n^2 + g'^2_n}$	23.9×10^6	704.0×10^6	$.1920 \times 10^6$	3729×10^6
$G_{n_1} = \sqrt{d_{n_1}^2 + d'^2_{n_1}}$	34.80×10^5	2990×10^5	$23{,}080 \times 10^5$	$85{,}250 \times 10^5$
$\psi_{n_1} = Q_{n_1}(H_{n_1} \div G_{n_1})$	$.5690 \times 10^{-4}$	$.240 \times 10^{-4}$	$.076 \times 10^{-4}$	$.0053 \times 10^{-4}$
$B_{n_1} \div A_{n_1} =$	$.251^{+}_{-}$	$.425^{+}_{+}$	$.0207^{+}_{+}$	$.451^{-}_{-}$
$X_{n_1} =$ arc tan B_{n_1}/A_{n_1}	165.9°	76.8°	1.2°	−155.7°
$g'_n \div g_n =$	4.67^{+}_{-}	$.139^{-}_{-}$	$.271^{-}_{-}$	$.387^{-}_{-}$
$Y_{n_1} =$ arc tan g'_n/g_n	102.1°	−172.1°	−164.8°	−158.8°
$d'_{n_1} \div d_{n_1} =$	$.0325^{+}_{-}$	$.037^{+}_{+}$	$.217^{+}_{+}$	$.345^{+}_{+}$
$Z_{n_1} =$ arc tan d'_{n_1}/d_{n_1}	178.2°	2.1°	12.3°	19.1°
$K_{n_1} = Y_{n_1} - Z_{n_1} + X_{n_1}$	89.8°	−97.4°	−175.9°	+333.6°

Amplitude of series $\psi_{n1} \sin(n\omega_1 t + K_{n_1}) = Y_1 = 1.295 \times 10^{-4}$.

† Reverse signs of torque effort in case of compressors.

PLATE III—*Continued.*

Power Pulsation in Three Synchronous Machines—Connected to Reciprocating Machinery—Machines No. 2.

Engine rating—6-cyl., 4-cycle, single acting, 16½″ × 24″. One cylinder missing.
Generator rating—36 p.—500 k.v.a.—400 k.w.—.8 p.f.—200 R.P.M.—2300 v.—3 φ—60~.

$P_{o_2} = 1370$	$\omega_2 = 10.47$	WR^2 of Mot. Gen. = 30,900
$T_2 = 5250$ H.P. ÷ R.P.M. $= .1407 \times 10^5$		WR^2 of Flywheel = 146,308
$T_{p_2} = T_2 P_{o_2} \times$ poles ÷ 2 k.w. $= 8.674 \times 10^5$		Total $WR^2 = 177,208$
$T_{d_2} = 8.674 \times 10^5$		$I_2 = WR^2 \div 32.2 = 5503$

n	1	2	3	4
$A_{n_2} = 100\ (a_{n_2} \div a_{o_1})$ †	−44.17	12.86	50.28	−6.051
$B_{n_2} = 100\ (b_{n_2} \div a_{o_1})$ †	11.08	54.63	1.042	−2.730
$E_{n_2} = \sqrt{A_{n_2}^2 + B_{n_2}^2}$	45.60	56.15	50.28	6.635
$Q_{n_2} = E_{n_2} \div I_2$	$.8286 \times 10^{-2}$	1.020×10^{-2}	$.9137 \times 10^{-2}$	$.1206 \times 10^{-2}$
$*d_{n_2} =$	-34.80×10^5	2990×10^5	$22,563 \times 10^5$	$80,468 \times 10^5$
$*d_{n_2}^1 =$	1.130×10^5	110.0×10^5	4898×10^5	$27,830 \times 10^5$
$n_2 =$	-61.24×10^6	-347.6×10^6	-824.9×10^6	-1493×10^6
$n_2^1 =$	-2.817×10^6	-65.382×10^6	-247.45×10^6	-608.44×10^6
$H_{n_2} = \sqrt{n_2^3 + n_2^{12}}$	61.24×10^6	352.5×10^6	858.5×10^6	1609×10^6
$G_{n_2} = \sqrt{d^2 + d^{12}}$	34.80×10^5	2990×10^5	$23,080 \times 10^5$	$85,250 \times 10^5$
$\psi_{n_2} = -Q_{n_2}(H_{n_2} \div G_{n_2}) \times P_1/P_2$	-1.458×10^{-4}	$-.1202 \times 10^{-4}$	$-.034 \times 10^{-4}$	$-.0023 \times 10^{-4}$
$B_{n_2} \div A_{n_2} =$				
$X_{n_2} =$ arc tan B_{n_2}/A_{n_2}	165.9°	76.8°	1.2°	−155.7°
$n_2^1 \div n_2 =$	.046 ⁻	.188 ⁻	.30 ⁻	.408 ⁻
$Y_{n_2} =$ arc tan n_2^1/n_2	−177.4°	−169.4°	−163.3°	−158°
$d_{n_2}^1 = d_{n_2} =$				
$Z_{n_2} =$ arc tan $d_{n_2}^1/d_{n_2}$	178.2°	2.1°	12.3°	19.1°
$K_{n_2} = Y_{n_2} - Z_{n_2} + X_{n_2}$	−189.7°	−94.7°	−174.4°	−333°

Amplitude of series $\psi_{n_2} \sin\ (n\omega_2 t + K_{n_2}) = Y_2 = 2.832 \times 10^{-4}$.

* From table for machine No. 2.
† Reverse signs of torque effort in case of compressors.

PLATE III—*Continued.*

Machines No. 3.

Engine rating—4-cyl., 4-cycle, single acting cranks 180°. One cylinder missing.

Generator rating—48 p.—625 k.v.a.—500 k.w.—.8 p.f.—150 R.P.M.—2300 v.—3 φ.—60~

$P_{o_3} = 1920$ | $\omega_3 = 7.85$	WR^2 of Gen. = 86,000
$T_3 = 5250$ H.P. ÷ R.P.M. $= .2345 \times 10^5$	WR^2 of Flywheel = 1,681,000
$T_{e_3} = T_3 P_{o_3} \times$ poles ÷ 2 k.w. $= 21.61 \times 10^5$	Total $WR^2 = 1{,}767{,}000$
$T_{d_3} = .2161 \times 10^5$	$I_3 = WR^2 \div 32.2 = 54{,}876$

n	1	2	3	4
$A_{n_3} = 100\ (a_{n_3} \div a_{o_1})$ † ...	−64.84	102.16	−54.34	−79.74
$B_{n_3} = 100\ (b_{n_3} \div a_{o_1})$ † ...	71.30	−10.69	−57.83	81.21
$E_{n_3} = \sqrt{A_{n_3}^2 + B_{n_3}^2}$	96.35	102.8	79.4	113.9
$Q_{n_3} = E_{n_3} \div I_3$	$.1756 \times 10^{-2}$	$.1873 \times 10^{-2}$	$.1447 \times 10^{-2}$	$.2076 \times 10^{-2}$
$d_{n_3} =$	70.28×10^5	429.5×10^5	5620×10^5	$22{,}528 \times 10^5$
$d_{n_3}{}^1 =$	34.94×10^5	-88.35×10^5	527.5×10^5	4888×10^5
‡$n_3 =$	2.055×10^5	-2.005×10^5	-8.77×10^5	-18.25×10^5
‡$n_3{}^1 =$	$.4310 \times 10^5$	$.233 \times 10^5$	-1.220×10^5	-4.550×10^5
$H_{n_3} = \sqrt{n_3{}^2 + n_3{}^{1^2}}$	2.1×10^5	2.02×10^5	8.85×10^5	15.95×10^5
$G_{n_3} = \sqrt{d_{n_3}{}^2 + d_{n_3}{}^{1^2}}$	78.5×10^5	439.0×10^5	5650×10^5	$23{,}040 \times 10^5$
$\psi_{n_3} = -Q_{n_3}(H_{n_3} \div G_{n_3}) \times P_1/P_3$	$-.3517 \times 10^{-4}$	$-.0646 \times 10^{-4}$	$-.017 \times 10^{-4}$	$-.0127 \times 10^{-4}$
$B_{n_3} \div A_{n_3} =$				
$X_{n_3} =$ arc tan B_{n_3}/A_{n_3} ...	−227.6°	−6.°	−133.2°	−225.5°
$n_3{}^1 \div n_3 =$	.2095 $\frac{+}{+}$	.1163 $\frac{+}{-}$	.139 $\frac{-}{-}$	.2496 $\frac{-}{-}$
$Y_{n_3} =$ arc tan $n_3{}^1/n_3$	11.9°	173.4°	−172.1°	−166°
$d_{n_3}{}^1 \div d_{n_3} =$	.497 $\frac{+}{+}$	.205 $\frac{-}{+}$	.094 $\frac{+}{+}$	.217 $\frac{+}{+}$
$Z_{n_3} =$ arc tan $d_{n_3}{}^1/d_{n_3}$	26.5°	−11.6°	5.4°	12.2°
$K_{n_3} = Y_{n_3} - Z_{n_3} + X_{n_3}$	−242.2°	179°	−310.7°	−43.7°

Amplitude of series $\psi_{n_3} \sin(n\omega_3 t + K_{n_3}) = Y_3 = .739 \times 10^{-4}$.

$$Y_1 + Y_2 + Y_3 = 4.866 \times 10^{-4}$$

Per cent. power pulsation $= T_{e_1}(Y_1 + Y_2 + Y_3) \dfrac{\text{I.H.P.}^* \times \text{p.f.}^* \times .746}{\text{k.w.}^*} = 442.1$ per cent.§

* Refers to machine No. 1.
† Reverse signs of torque effort in case of compressors.
‡ From table for machine No. 3, page 795.
§ Percentage based on k.v.a. rating of generator No. 1.

rating of the six-cylinder engine. In this case there are no impressed forces having a frequency equal to half the revolutions. Under this ideal condition, parallel operation would be perfect. It is a fact, however, that such ideal conditions seldom exist. In multicylinder engines, even though a cylinder may not skip completely, it is a difficult matter to adjust the fuel supply to the different cylinders so uniformly that all give exactly the same amount of power. The result is that there is an impressed force, the frequency of which is equal to one-half the number of revolutions.

As a basis for making flywheel calculations for multicylinder engines, it would be reasonable to assume that one cylinder gives 20 per cent. less power than the others. This would probably represent the worst operating condition likely to be met with in practice. Flywheels designed to limit the power pulsation to 60 per cent. under this condition of operation would certainly give perfect parallel operation under all ordinary conditions.

No doubt at some future time, manufacturers of engine-driven generator units will be asked to guarantee the power pulsation, and it will be necessary to establish some such assumption as the above as a standard basis for making calculations of power pulsation.

POWER PULSATIONS IN FOUR SYNCHRONOUS MACHINES (NEGLECTING DAMPING).

If the flywheels used on parallel connected synchronous machines are properly designed, excessive values of power pulsation will not occur. Resonance effects will be reduced to a minimum and the amortisseur windings will have but little effect on the magnitude of the power pulsation. An approximate value of the pulsation may therefore be determined by neglecting the damping torques in the fundamental equations. This simplifies the work so much that the case of four machines, neglecting damping, becomes rather simpler than the case of three machines, just worked out, in which damping was taken into consideration.

Neglecting the T_d terms, the five general equations governing the hunting of four machines may be written down directly by analogy with the equations for three machines.

$$\psi_1\left(\frac{T_{s_1}}{I_1}+p^2\right)+\frac{P_1}{P_1}p^2\varphi_1=\frac{1}{I_1}f_1(t) \tag{1}$$

$$\psi_2\left(\frac{T_{s_2}}{I_2}+p^2\right)+\frac{P_1}{P_2}p^2\varphi_1=\frac{1}{I_2}f_2(t) \tag{2}$$

$$\psi_3\left(\frac{T_{s_3}}{I_3}+p^2\right)+\frac{P_1}{P_3}p^2\varphi_1=\frac{1}{I_3}f_3(t) \tag{3}$$

$$\psi_4\left(\frac{T_{s_4}}{I_4}+p^2\right)+\frac{P_1}{P_4}p^2\varphi_1=\frac{1}{I_4}f_4(t) \tag{4}$$

$$T_{s_1}\frac{P_1}{P_1}\psi_1+T_{s_2}\frac{P_1}{P_2}\psi_2+T_{s_3}\frac{P_1}{P_3}\psi_3+T_{s_4}\frac{P_1}{P_4}\psi_4=0 \tag{5}$$

In the above equation let

$$\left.\begin{aligned} A_n&=\frac{T_{sn}}{I_n}+p^2=\alpha_n+p^2,\ B_n=\frac{P_1}{P_n}p^2\\ C_n&=T_{sn}\frac{P_1}{P_n},\ F_n=\frac{1}{I_n}f_n(t)\end{aligned}\right\} \tag{6}$$

Making these substitutions, the above equations become

$$\left.\begin{aligned} &\psi_1A_1+B_1\varphi_1=F_1\\ &\psi_2A_2+B_2\varphi_1=F_2\\ &\psi_3A_3+B_3\varphi_1=F_3\\ &\psi_4A_4+B_4\varphi_1=F_4\\ &C_1\psi_1+C_2\psi_2+C_3\psi_3+C_4\psi_4=0\end{aligned}\right\} \tag{7}$$

The denominator determinant is

$$\begin{vmatrix} A_1 & 0 & 0 & 0 & B_1\\ 0 & A_2 & 0 & 0 & B_2\\ 0 & 0 & A_3 & 0 & B_3\\ 0 & 0 & 0 & A_4 & B_4\\ C_1 & C_2 & C_3 & C_4 & 0\end{vmatrix} = -\left\{\begin{array}{l} B_1C_1A_2A_3A_4\\ +B_2C_2A_1A_3A_4\\ +B_3C_3A_1A_2A_4\\ +B_4C_4A_1A_2A_3\end{array}\right\} \tag{8}$$

For three machines the denominator would have been

$$D=-\left\{\begin{array}{l} B_1C_1A_2A_3\\ +B_2C_2A_1A_3\\ +B_3C_3A_1A_2\end{array}\right\}$$

For two it would have been

$$D=-\left\{\begin{array}{l} B_1C_1A_2\\ +B_2C_2A_1\end{array}\right\}.$$

The law, by which the denominator for any number of machines is formed, is now evident and could be written down

at once. For each additional machine there is one more term and one more A in each term.

Now $B_n = \frac{P_1}{P_n}\, p^2$ and $C_n = T_{3n}\frac{P_1}{P_n}$ so that the denominator for four machines becomes

$$D = -p^2\left\{\begin{array}{l} T_{3_1}\left(\frac{P_1}{P_1}\right)^2 A_2A_3A_4 \\ +\, T_{3_2}\left(\frac{P_1}{P_2}\right)^2 A_1A_3A_4 \\ +\, T_{3_3}\left(\frac{P_1}{P_3}\right)^2 A_1A_2A_4 \\ +\, T_{3_4}\left(\frac{P_1}{P_4}\right)^2 A_1A_2A_3 \end{array}\right\} = -p^2\left\{\begin{array}{l} \beta_1A_2A_3A_4 \\ +\,\beta_2A_1A_3A_4 \\ +\,\beta_3A_1A_2A_4 \\ +\,\beta_4A_1A_2A_3 \end{array}\right\} \quad (9)$$

where

$$\beta_n = T_{3n}\left(\frac{P_1}{P_n}\right)^2.$$

The products $A_2A_3A_4$, $A_1A_3A_4$, etc., are all of the same general form since $A_n = (\alpha_n + p^2)$. Consider a product of the type $A_1A_2A_3$

$$= \alpha_1\alpha_2\alpha_3 + p^2(\alpha_1\alpha_3 + \alpha_2\alpha_3 + \alpha_1\alpha_2) + p^4(\alpha_1 + \alpha_2 + \alpha_3) + p^6.$$

The coefficient of the highest power, p^6, is 1, while that of the next highest, p^4, consists of a sum of all the combinations of the α terms involved, taken one at a time. Similarly the coefficient of the next highest power, p^2, consists of a sum of the combinations of α_n taken two at a time.

Following the same scheme the coefficients for a product of any number of terms could be written down at once.

Hence for four machines

$$D = -p^2[M_1p^6 + M_2p^4 + M_3p^2 + M_4] \quad (10)$$

in which

$$M_1 = [\beta_1 + \beta_2 + \beta_3 + \beta_4] \quad (11)$$

$$M_2 = \beta_1\left[\begin{array}{l}\text{Combinations of } \alpha_n \text{ taken one at a time excepting terms} \\ \text{containing } \alpha_1\end{array}\right]$$
$$+\,\beta_2\left[\text{Similar combinations excepting terms containing } \alpha_2\right]$$
$$+\,\beta_3\left[\text{Similar combinations excepting terms containing } \alpha_3\right]$$
$$+\,\beta_4\left[\text{Similar combinations excepting terms containing } \alpha_4\right] \quad (12)$$

$$M_3 = \beta_1\Big[\text{Combinations of } \alpha_n \text{ taken two at a time excepting terms containing } \alpha_1\Big] + \beta_2\Big[\text{Same, excepting terms containing } \alpha_2\Big] + \beta_3\Big[\text{Same, excepting terms containing } \alpha_3\Big] + \beta_4\Big[\text{Same, excepting terms containing } \alpha_4\Big] \quad (13)$$

$$M_4 = \beta_1\Big[\;\Big] + \beta_2\Big[\;\Big] + \beta_3\Big[\;\Big] + \beta_4\Big[\;\Big] \quad (14)$$

(Combinations of α_n taken three at a time with exceptions as above.)

It will be found that the $-p^2$ coefficient in equation (10) will cancel with a similar term in the numerator.

The value of the denominator must be calculated for each harmonic in each of the torque curves. Hence for the case of four machines under discussion, the denominator will have to be evaluated for sixteen different values of frequency. To calculate D for any given frequency it is, of course, only necessary to put $p = j\omega$ and calculate the bracket in (10). The M's are constants for any particular combination of machines.

The numerator determinant for ψ_1 is

$$\begin{vmatrix} F_1 & 0 & 0 & 0 & B_1 \\ F_2 & A_2 & 0 & 0 & B_2 \\ F_3 & 0 & A_3 & 0 & B_3 \\ F_4 & 0 & 0 & A_4 & B_4 \\ 0 & C_2 & C_3 & C_4 & 0 \end{vmatrix} \quad (15)$$

The coefficient of F_1 in this determinant is found to be

$$-p^2[N_1 p^4 + N_2 p^2 + N_3] \quad (16)$$

where

$$\left.\begin{aligned} N_1 &= \beta_2 + \beta_3 + \beta_4 \\ N_2 &= \beta_2(\alpha_3 + \alpha_4) + \beta_3(\alpha_2 + \alpha_4) + \beta_4(\alpha_2 + \alpha_3) \\ N_3 &= \beta_2(\alpha_3\alpha_4) + \beta_3(\alpha_2\alpha_4) + \beta_4(\alpha_2\alpha_3) \end{aligned}\right\} \quad (17)$$

The law governing the coefficient of F_1 for any number of machines is fairly evident and a general expression for n machines could be written down at once.

The coefficient of F_2 in the numerator is

$$B_1 C_2 A_3 A_4 = p^2 \left[T_{s_2} \frac{P_1}{P_2} (p^4 + (\alpha_3 + \alpha_4) p^2 + \alpha_3 \alpha_4) \right] \tag{18}$$

Similarly the coefficient of F_3 is

$$B_1 C_4 A_2 A_3 = p^2 \left[T_{s_4} \frac{P_1}{P_4} (p^4 + (\alpha_2 + \alpha_3) p^2 + \alpha_2 + \alpha_3) \right] \tag{19}$$

A term $(-p^2)$ will cancel in all the terms of the numerator with the same coefficient in the denominator.

To get the numerator corresponding to ψ_2, it is only necessary to interchange subscripts 1 and 2 in equations (17), (18) and (19). In a similar manner the numerator corresponding to any ψ can be found.

It must be remembered that the F's themselves are Fourier's series of four harmonics so that in equation (20) there are really four complete Fourier's series; and in a system of four machines, sixteen Fourier's series would be required in all, to get the pulsations in all the machines.

Small Terms May be Neglected.—In any actual problem where the resonance frequencies are far below all the impressed harmonics, and this is the only condition to which the above theory is applicable, certain terms are very small and may be neglected without appreciably affecting the result. For instance, N_3 is very small and may be neglected in coefficient of F_1. Likewise the products $\alpha_3 \alpha_4$, $\alpha_2 \alpha_4$ and $\alpha_2 \alpha_3$ may be neglected in the other terms in the numerator. M_4 may also be neglected in the denominator. This allows another p^2 to cancel from the numerator and denominator and gives:

$$\psi_1 = \left\{ (N_1 p^3 + N_2) F_1 - T_{s_2} \frac{P_1}{P_2} [p_3 + (\alpha_3 + \alpha_4)] F_2 - T_{s_3} \frac{P_1}{P_3} [p^3 + (\alpha_2 + \alpha_4)] F_3 \right.$$
$$\left. - T_{s_4} \frac{P_1}{P_4} [p^3 + (\alpha_2 + \alpha_3)] F_4 \right\} \div (M_1 p^4 + M_2 p + M_3) \tag{20}$$

Putting $p = j\omega$,

$$\psi_1 = \left\{ (N_2 - N_1 \omega^2) F_1 - T_{s_2} \frac{P_1}{P_2} [(\alpha_3 + \alpha_4) - \omega^2] F_2 - T_{s_3} \frac{P_1}{P_3} [(\alpha_2 + \alpha_4) - \omega^2] F_3 \right.$$
$$\left. - T_{s_4} \frac{P_1}{P_4} [(\alpha_2 + \alpha_3) - \omega^2] F_4 \right\} \div (M_1 \omega^4 - M_2 \omega^2 + M_3) \tag{21}$$

If Y_o is the sum of the amplitudes of the four Fourier's

series represented in equation (21), the percentage power pulsation of machine No. 1, based on its own k.v.a. rating, is

$$Y_0 \left(\frac{T_{s_1} S \times 100}{7040 \text{ k.v.a}} \right) \text{ per cent.} \qquad (22)$$

As an example illustrating the above theory, consider the calculation of the power pulsation between the following four engine generator units:

Unit No. 1.—Same as unit No. 1, page 618.
Unit No. 2.—Same as unit No. 1, page 618.
Unit No. 3.—Same as unit No. 2, page 618.
Unit No. 4.

Generator.—32 pole—150 k.v.a.—120 k.w.—.8 p.f.—225 R.P.M.—2300 volt—3 phase—60 cycle. $P_0 = 254$.

Engine.—2-cylinder, single acting, 4-cycle, cranks at 360°.

The WR^2 of the generator rotor is 7640 and that of the engine flywheel is 270,000. The Fourier's series representing the variations in the engine torque is:

$$f(t) = 221\,[98 \sin\theta - 53 \sin 2\theta + 12 \sin 3\theta + 26 \sin 4\theta + 26 \cos\theta - 12 \cos 2\theta - 13 \cos 3\theta - 12 \cos 4\theta].$$

The necessary calculations are shown in Plate IV. It should be noted that while this calculation may seem very long, the pulsation in each one of the four machines has been calculated. It may also be of interest to know that this calculation, as well as the others contained in this paper, was made by high-school girls who knew nothing of the theory involved but who were familiar with the operation of calculating machines.

The calculations give the following values of power pulsation:

Unit No. 1	No. 2	No. 3	No. 4
94 per cent.	94 per cent.	66.9 per cent.	58.8 per cent.

These values are based on the k.w. ratings of the machines. On the basis of the k.v.a. ratings the percentages would be 80 per cent. of those given. It should be remembered that these results represent the worst possible condition of power pulsation since units No. 1 and No. 2 are identical and the power pulsation of these units can be reduced by changing the angle between the cranks. One would therefore conclude that these units were equipped with flywheels which would give satisfactory parallel operation under ordinary conditions.

RESONANCE FREQUENCIES IN PARALLEL CONNECTED SYNCHRONOUS MACHINES.

Resonance in a mechanical system is almost identical with resonance in an electrical system, though most engineers are more familiar with the phenomenon in connection with the latter.

If a condenser is discharged through an inductance coil L having r ohms resistance, the frequency of the oscillation is known to be:

$$\omega = \sqrt{\frac{1}{LC} - \frac{r^2}{4L^2}}\ ,^9$$

This is called the *natural* frequency and it is the frequency at which the circuit oscillates when left entirely to itself.

If, however, a voltage is impressed on a series circuit of inductance, resistance and capacity, and the frequency of the

PLATE IV.

Power Pulsations in Four Synchronous Machines—Neglecting Damping.

Machine No. 1—Rating—40 pole—250 k.v.a.—200 k.w.—.8 p.f.—180 R.P.M.—2300 v.

$P_0 = 723$ Total $WR^2 = 752{,}600$ $I_1 = 2.337 \times 10^4$

$T_{s_1} = 3520\left(\frac{P_0 P_1}{S}\right) = 56.52 \times 10^4$, $\alpha_1 = \frac{T_{s_1}}{I_1} = 24.2$ $\beta_1 = T_{s_1}\left(\frac{P_1}{P_1}\right)^2 = 56.52 \times 10^4$

Machine No. 2—Rating—Same as No. 1.

$P_0 = 723$ Total $WR^2 = 752{,}600$ $I_2 = 2.337 \times 10^4$

$T_{s_2} = 3520\left(\frac{P_0 P_2}{S}\right) = 56.52 \times 10^4$, $\alpha_2 = \frac{T_{s_2}}{I_2} = 24.2$ $\beta_2 = T_{s_2}\left(\frac{P_1}{P_2}\right)^2 = 56.52 \times 10^4$

Machine No. 3—Rating—28 pole—120 k.v.a.—96½ k.w.—.8 p.f.—257 R.P.M.—2300 v.

$P_0 = 305$ Total $WR^2 = 168{,}900$ $I_3 = 5.245 \times 10^3$

$T_{s_3} = 3520\left(\frac{P_0 P_3}{S}\right) = 11.70 \times 10^4$, $\alpha_3 = \frac{T_{s_3}}{I_3} = 22.3$ $\beta_3 = T_{s_3}\left(\frac{P_1}{P_3}\right)^2 = 23.85 \times 10^4$

Machine No. 4—Rating—32 pole—150 k.v.a.—120 k.w.—.8 p.f.—225 R.P.M.—2300 v.

$P_0 = 254$ Total $WR^2 = 277{,}640$ $I_4 = 8.622 \times 10^3$

$T_{s_4} = 3520\left(\frac{P_0 P_4}{S}\right) = 12.71 \times 10^4$, $\alpha_4 = \frac{T_{s_4}}{I_4} = 14.75$ $\beta_4 = T_{s_4}\left(\frac{P_1}{P_4}\right)^2 = 19.85 \times 10^4$

[9] This is given in radians per second, so that the number of cycles per second is $\frac{\omega}{2\pi}$.

PLATE IV—*Continued.*

Sec. 1.

n	1	2	3	4
$f_1(t)$ a_n	38514	−20829	4716	10218
$f_1(t)$ b_n	10218	−4716	−5109	−4716
$Q_n = \sqrt{a_n^2 + b_n^2}$	.39800	21350	6955	11260
$F_{n_1} = Q_n / I_1$	1.703	.9138	.2977	4819
$X_n = \tan^{-1} b_n / a_n$	14.9°	193°	313°	335°

Sec. 2.

$f_2(t)$ a_n	38514	−20829	4716	10218
$f_2(t)$ b_n	10218	−4716	−5109	−4716
$Q_n = \sqrt{a_n^2 + b_n^2}$	.39800	21350	6955	11260
$F_{n_2} = Q_n / I_2$	1.703	.9138	.2977	4819
$X_n = \tan^{-1} b_n / a_n$	14.9°	193°	313°	335°

Sec. 3.

$f_3(t)$ a_n	14896	−8056	1824	3952
$f_3(t)$ b_n	3952	−1824	1976	−1824
$Q_n = \sqrt{a_n^2 + b_n^2}$	1.5400	8260	2690	4350
$F_{n_3} = Q_n / I_3$	2.937	1.575	.5130	.8295
$X_n = \tan^{-1} b_n / a_n$	14.9°	193°	313°	335°

Sec. 4.

$f_4(t)$ a_n	21658	−11713	2652	5746
$f_4(t)$ b_n	5746	−2652	−2873	−2652
$Q_n = \sqrt{a_n^2 + b_n^2}$	22390	12010	3910	6340
$F_{n_4} = Q_n / I_4$	2.597	1.393	4536	.7354
$X_n = \tan^{-1} b_n / a_n$	14.9°	193°	313°	335°

PLATE IV—*Continued.*

$\alpha_1 = 24.2$	$\alpha_1+\alpha_2+\alpha_3 = 70.7$ ①	$\alpha_2\alpha_3+\alpha_2\alpha_4+\alpha_3\alpha_4 = 1225.6$ ⑤
$\alpha_2 = 24.2$	$\alpha_1+\alpha_2+\alpha_4 = 63.15$ ②	$\alpha_1\alpha_3+\alpha_1\alpha_4+\alpha_3\alpha_4 = 1225.6$ ⑥
$\alpha_3 = 22.3$	$\alpha_1+\alpha_3+\alpha_4 = 61.25$ ③	$\alpha_1\alpha_2+\alpha_1\alpha_4+\alpha_3\alpha_4 = 1299.6$ ⑦
$\alpha_4 = 14.75$	$\alpha_2+\alpha_3+\alpha_4 = 61.25$ ④	$\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_3\alpha_3 = 1665$ ⑧
$\alpha_1\alpha_2 = 585.6$	$\beta_1 = 56.52\times10^4$	$M_2 = \beta_1\times④+\beta_2\times⑤+\beta_3\times②+\beta_4\times①=.983\times10^6$
$\alpha_1\alpha_3 = 539.7$	$\beta_2 = 56.52\times10^4$	
$\alpha_1\alpha_4 = 357.0$	$\beta_3 = 23.85\times10^4$	$M_3 = \beta_1\times⑤+\beta_2\times⑥+\beta_3\times⑦+\beta_4\times⑧=2.026\times10^9$
$\alpha_2\alpha_3 = 539.7$	$\beta_4 = 19.85\times10^4$	
$\alpha_2\alpha_4 = 357.0$	$\Sigma\beta = M_1 = 1.567\times10^4$	
$\alpha_3\alpha_4 = 328.9$		

	n	$n\omega$	$n^2\omega^2$	$n^4\omega^4$	$M_1n^4\omega^4$	⑨ = $M_1n^4\omega^4+M_2$	⑩ = $M_3n^2\omega^2$	D = ⑨ − ⑩
1st Machine	1	18.85	3.55×10^2	1.260×10^5	1.974×10^{11}	1.994×10^{11}	3.49×10^{10}	1.645×10^{11}
	2	37.70	1.421×10^3	2.016×10^6	3.159×10^{12}	3.161×10^{12}	1.396×10^{11}	3.021×10^{12}
	3	56.55	3.20×10^3	1.021×10^7	1.600×10^{13}	1.600×10^{13}	3.15×10^{11}	1.564×10^{13}
	4	75.40	5.69×10^3	3.23×10^7	5.06×10^{13}	5.06×10^{13}	5.59×10^{11}	5.01×10^{13}
2nd Machine	1	18.85	3.55×10^2	1.260×10^5	1.974×10^{11}	1.994×10^{11}	3.49×10^{10}	1.645×10^{11}
	2	37.70	1.421×10^3	2.016×10^6	3.159×10^{12}	3.161×10^{12}	1.396×10^{11}	3.021×10^{12}
	3	56.55	3.20×10^3	1.021×10^7	1.600×10^{13}	1.600×10^{13}	3.15×10^{11}	1.564×10^{13}
	4	75.40	5.69×10^3	3.23×10^7	5.06×10^{13}	5.06×10^{13}	5.59×10^{11}	5.01×10^{13}
3d Machine	1	26.90	7.24×10^2	5.24×10^5	8.21×10^{11}	8.23×10^{11}	7.12×10^{10}	7.52×10^{11}
	2	53.8	2.90×10^3	8.38×10^6	13.13×10^{12}	13.13×10^{12}	2.85×10^{11}	12.84×10^{12}
	3	80.7	6.52×10^3	4.24×10^7	6.64×10^{13}	6.64×10^{13}	6.41×10^{11}	6.58×10^{13}
	4	107.6	11.58×10^3	13.41×10^7	21.01×10^{13}	21.01×10^{13}	11.38×10^{11}	20.90×10^{13}
4th Machine	1	23.56	5.55×10^2	3.08×10^5	4.83×10^{11}	4.85×10^{11}	5.46×10^{10}	4.30×10^{11}
	2	47.12	2.22×10^3	4.93×10^6	7.73×10^{12}	7.73×10^{12}	2.18×10^{11}	7.51×10^{12}
	3	70.68	4.99×10^3	2.49×10^7	3.90×10^{13}	3.90×10^{13}	4.91×10^{11}	3.85×10^{13}
	4	94.24	8.88×10^3	7.88×10^7	12.35×10^{13}	12.35×10^{13}	8.73×10^{11}	12.26×10^{13}

PLATE IV—*Continued.*

Hunting of Machine No. 1.

Due to Machine No. 1

$N_1=\beta_2+\beta_3+\beta_4=100.2\times10^4$

$N_2=\beta_2(\alpha_3+\alpha_4)+\beta_3(\alpha_2+\alpha_4)+\beta_4(\alpha_2+\alpha_3)=3.946\times10^7$

$n^2\omega_1^2$	3.55×10^2	1.42×10^3	3.20×10^3	5.69×10^3
① $N_1n^2\omega_1^2$	3.55×10^6	1.42×10^7	3.20×10^7	5.69×10^7
② N_2	$.395\times10^8$			
$G_1=[②-①]$	-3.155×10^8	-1.38×10^7	-3.16×10^7	-5.65×10^7
$\psi_n=G_1F_1\div D$	-3.27×10^{-3}	$-.417\times10^{-3}$	$-.6\times10^{-4}$	$-.545\times10^{-4}$

Amp. $=Y_1=6.67\times10^{-3}$. For phase angles see page 806, sec. 1.

Due to Machine No. 2

⑥ $n^2\omega_2^2$	3.55×10^2	14.21×10^2	32.0×10^2	56.9×10^2
⑦ $(\alpha_3+\alpha_4)$	$.37\times10^2$	$.57\times10^2$	$.37\times10^2$	$.37\times10^2$
⑧ $[⑦-⑥]$	-3.18×10^2	-13.84×10^2	-31.63×10^2	-56.53×10^2
$G_2=T_{s_2}[P_1/P_2]$⑧	-779.7×10^6	-782.2×10^6	-1788×10^6	-3195×10^6
$\psi_n=G_2F_2\div D$	1.860×10^{-4}	$-.237\times10^{-3}$	$-.0339\times10^{-3}$	$-.0308\times10^{-3}$

Amp. $=Y_2=4.32\times10^{-4}$. For phase angles see page 806, sec. 2.

Due to Machine No. 3

⑨ $n^2\omega_3^2$	7.24×10^2	29.0×10^2	65.2×10^2	115.8×10^2
⑩ $(\alpha_2+\alpha_4)$	$.39\times10^2$	$.39\times10^2$	$.39\times10^2$	$.39\times10^2$
⑪ ④ − ③	-6.85×10^2	-28.61×10^2	-64.81×10^2	-115.4×10^2
$G_3=T_{s_3}[P_1/P_3]$⑤	-114.5×10^6	-478.4×10^6	-1084×10^6	-1929×10^6
$\psi_n=G_3F_3\div D$	-44.73×10^{-6}	-5.868×10^{-6}	$-.843\times10^{-6}$	$-.762\times10^{-6}$

Amp. $=Y_3=90.90\times10^{-6}$. For phase angles see page 806, sec. 3.

Due to Machine No. 4

⑫ $n^2\omega_4^2$	5.55×10^2	22.2×10^2	49.9×10^2	88.8×10^2
⑬ $(\alpha_2+\alpha_3)$	$.46\times10^2$	$.46\times10^2$	$.46\times10^2$	$.46\times10^2$
⑭ $[⑬-⑫]$	-5.09×10^2	-21.74×10^2	-4944×10^2	-88.34×10^2
$G_4=T_{s_4}[P_1/P_4]$⑪	80.88×10^6	-345.4×10^6	-785.6×10^6	1404×10^6
$\psi_n=G_4F_4\div D$	48.85×10^{-5}	-6.41×10^{-6}	$-.926\times10^{-6}$	$-.843\times10^{-6}$

Amp. $=Y_4=114.1\times10^{-6}$. For phase angles see page 806, sec. 4.

$Y_0=Y_1+Y_2+Y_3+Y_4=13.04\times10^{-3}$

Per cent. power pulsation $=Y_0\left(\frac{T_{s_1}\ S\times100}{7040\text{ k.w.}}\right)=94$ per cent.

PLATE IV—*Continued.*

Hunting of Machine No. 2.

Due to Machine No. 2

$N_1=\beta_1+\beta_2+\beta_4=100.2\times10^4$

$N_2=\beta_1(\alpha_2+\alpha_4)+\beta_2(\alpha_1+\alpha_4)+\beta_4(\alpha_1+\alpha_2)=3.946\times10^7$

$n^2\omega_2^2$	3.55×10^2	14.21×10^2	32.0×10^2	56.9×10^2
① $N_1n^2\omega_2^2$	355.7×10^6	1424×10^6	3206×10^6	5701×10^6
② N_2	39.5×10^6	39.5×10^6	39.5×10^6	39.5×10^6
$G_2=$[②−①]	-316.2×10^6	-1384×10^6	-3167×10^6	-5661×10^6
$\psi_n=G_2F_2\div D$	-3.27×10^{-3}	-419×10^{-3}	-601×10^{-3}	-546×10^{-3}

Amp. $=Y_2=7.61\times10^{-3}$ For phase angles see page 806, sec. 2.

Due to Machine No. 1

⑥ $n^2\omega_1^2$	3.55×10^2	14.21×10^2	32.0×10^2	56.9×10^2
⑦ $(\alpha_2+\alpha_4)$	$.37\times10^2$	$.37\times10^2$	$.37\times10^2$	$.37\times10^2$
⑧[⑦−⑥]	-3.18×10^2	-13.821×10^2	-31.63×10^2	-56.53×10^2
$G_1=T_{s_1}[P_2/P_1]$⑧	-179.7×10^6	-782.2×10^6	-1788×10^6	-3195×10^6
$\psi_n=G_1F_1\div D$	-1.860×10^{-3}	$-.237\times10^{-3}$	$-.0339\times10^{-3}$	$-.0308\times10^{-3}$

Amp. $=Y_1=4.32\times10^{-3}$. For phase angles see page 806, sec. 1.

Due to Machine No. 3

③ $n^2\omega_3^2$	7.04×10^2	29.0×10^2	65.2×10^2	118.8×10^2
⑤ $(\alpha_1+\alpha_4)$	$.39\times10^2$	$.39\times10^2$	$.39\times10^2$	$.39\times10^2$
⑨[④−③]	-6.85×10^2	-28.61×10^2	-6481×10^2	-115.4×10^2
$G_3=T_{s_3}[P_2/P_3]$⑤	-114.5×10^6	-478.4×10^6	-1084×10^6	-1929×10^6
$\psi_n=G_3F_3\div D$	-44.73×10^{-5}	-5.868×10^{-5}	$-.843\times10^{-5}$	$-.762\times10^{-5}$

Amp. $=Y_3=90.9\times10^{-6}$. For phase angles see page 806, sec. 3.

Due to Machine No. 4

④ $n^2\omega_4^2$	5.55×10^2	22.2×10^2	49.9×10^2	88.8×10^2
⑩ $(\alpha_1+\alpha_3)$	$.46\times10^2$	$.46\times10^2$	$.46\times10^2$	$.46\times10^2$
⑪[⑩−⑨]	-5.09×10^2	-21.74×10^2	-49.44×10^2	-88.34×10^2
$G_4=T_{s_4}[P_2/P_4]$⑪	-80.88×10^6	-345.4×10^6	-785.6×10^6	-1404×10^6
$\psi_n=G_4F_4\div D$	-48.85×10^{-5}	-6.41×10^{-5}	$-.926\times10^{-5}$	-842×10^{-5}

Amp. $=Y_4=114.1\times10^{-6}$. For phase angles see page 806, sec. 4.

$Y_o=Y_1+Y_2+Y_3+Y_4=14.115\times10^{-3}$

Per cent. power pulsation $=Y_o\left(\frac{T_{s_2}\ S\times100}{7040\text{ k.w.}}\right)=94$ per cent.

PLATE IV—*Continued.*
Hunting of Machine No. 3.

Due to Machine No. 3

$N_1=\beta_2+\beta_1+\beta_4=132.9\times10^4$

$N_2=\beta_2(\alpha_1+\alpha_4)+\beta_1(\alpha_2+\alpha_4)+\beta_4(\alpha_2+\alpha_1)=5.364\times10^7$

$n^2\omega_3^2$	7.24×10^2	29.0×10^2	65.2×10^2	115.8×10^2
① $N_1n^2\omega_3^2$	9.622×10^8	38.54×10^8	86.65×10^8	153.9×10^8
② N_2	$.536\times10^8$	$.536\times10^8$	$.536\times10^8$	$.536\times10^8$
$G_3=$[②−①]	-9.086×10^8	-38.00×10^8	-86.11×10^8	-153.4×10^8
$\psi_n=G_3F_3+D\ldots$	-3.549×10^{-3}	$.466\times10^{-3}$	$.067\times10^{-3}$	$.061\times10^{-3}$

Amp. $=Y_3=8.29\times10^{-3}$. For phase angles see page 806, sec. 3.

Due to Machine No. 2

⑥ $n^2\omega_2^2$	3.55×10^2	14.21×10^2	32.0×10^2	56.9×10^2
⑦ $(\alpha_1+\alpha_4)$	$.39\times10^2$	$.39\times10^2$	$.39\times10^2$	$.39\times10^2$
⑧[⑦−⑥]	-3.16×10^2	-13.82×10^2	-31.61×10^2	-5651×10^2
$G_2=T_{s_2}[P_2/P_1]$⑧	-125.0×10^6	-546.8×10^6	-1251×10^6	-2236×10^6
$\psi_n=G_2F_2+D\ldots$	-129.4×10^{-6}	-16.57×10^{-6}	-2.377×10^{-6}	-2.155×10^{-6}

Amp. $=Y_2=301.0\times10^{-6}$. For phase angles see page 806, sec. 2.

Due to Machine No. 1

⑤ $n^2\omega_1^2$	3.55×10^2	14.21×10^2	32.00×10^2	56.9×10^2
④ $(\alpha_2+\alpha_4)$	$.39\times10^2$	$.39\times10^2$	$.39\times10^2$	$.39\times10^2$
⑨[④−③]	-3.16×10^2	-13.82×10^2	31.61×10^2	-56.51×10^2
$G_1=T_{s_1}[P_1/P_1]$⑤	-125.0×10^6	-546.8×10^6	-1251×10^6	-2236×10^6
$\psi_n=G_1F_1+D\ldots$	-129.4×10^{-6}	-16.57×10^{-6}	-2.375×10^{-6}	-2.155×10^{-6}

Amp. $=Y_1=301.0\times10^{-6}$. For phase angles see page 806, sec. 1.

Due to Machine No. 4

⑨ $n^2\omega_4^2$	5.55×10^2	22.2×10^2	49.9×10^2	888×10^2
⑩ $(\alpha_2+\alpha_1)$	$.48\times10^2$	$.48\times10^2$	$.48\times10^2$	$.48\times10^2$
⑪[⑩−⑨]	-5.07×10^2	-21.72×10^2	29.42×10^2	-88.32×10^2
$G_4=T_{s_4}[P_2/P_4]$⑪	-56.38×10^6	241.5×10^6	-549.6×10^6	-982.2×10^6
$\psi_n=G_4F_4+D\ldots$	-34.05×10^{-6}	-4.479×10^{-6}	$.647\times10^{-6}$	$-.589\times10^{-6}$

Amp. $=Y_4=79.5\times10^{-6}$. For phase angles see page 806, sec. 4.

$Y_o=Y_1+Y_2+Y_3+Y_4=15.105\times10^{-3}$

Per cent. power pulsation $=Y_o\left(\frac{T_{s_3}\ S\times100}{7040\text{ k.w.}}\right)=66.9$ per cent.

PLATE IV—*Continued.*

Hunting of Machine No. 4.

Due to Machine No. 4

$N_1=\beta_2+\beta_3+\beta_1=136.9\times10^4$

$N_2=\beta_2(\alpha_3+\alpha_1)+\beta_3(\alpha_2+\alpha_1)+\beta_1(\alpha_2+\alpha_3)=6.411\times10^7$.

$n^2\omega_4{}^2$	5.55×10^2	22.2×10^2	49.9×10^2	88.8×10^2
① $N_1n^2\omega_4{}^2$	7.598×10^2	30.39×10^8	68.31×10^8	121.6×10^8
② N_2	$.641\times10^8$	$.641\times10^8$	$.641\times10^8$	$.641\times10^8$
$G_4=[②-①]$	-6.957×10^8	-29.95×10^8	-67.67×10^8	-121.1×10^8
$\psi_n=G_4F_4\div D$	-4.202×10^{-3}	$.552\times10^{-3}$	$-.080\times10^{-3}$	$-.073\times10^{-3}$

Amp. $=Y_4=9.81\times10^{-3}$. For phase angles see page 806, sec. 4.

Due to Machine No. 2

④ $n^2\omega_2{}^2$	3.55×10^2	14.21×10^2	32.0×10^2	56.9×10^2
⑦ $(\alpha_3+\alpha_1)$	$.46\times10^2$	$.46\times10^2$	$.46\times10^2$	$.46\times10^2$
⑧ $[⑦-⑥]$	-3.09×10^2	-13.75×10^2	31.54×10^2	56.44×10^2
$G_2=T_{s_2}[P_4/P_2]⑧$	-139.7×10^6	-621.7×10^6	-1426×10^6	-2552×10^6
$\psi_n=G_2F_2\div D$	-144.6×10^{-6}	-18.81×10^{-5}	-2.709×10^{-5}	-2.460×10^{-5}

Amp. $=Y_2=337.2\times10^{-6}$. For phase angles see page 806, sec. 2.

Due to Machine No. 3

⑤ $n^2\omega_3{}^2$	7.24×10^2	29.0×10^2	65.2×10^2	115.8×10^2
⑥ $(\alpha_2+\alpha_1)$	$.48\times10^2$	$.48\times10^2$	$.48\times10^2$	$.48\times10^2$
⑨ $[⑥-③]$	-6.76×10^2	-28.52×10^2	-64.72×10^2	-115.3×10^2
$G_3=T_{s_2}[P_4/P_3]⑨$	-90.40×10^6	-381.4×10^6	-865.5×10^6	-1542×10^6
$\psi_n=G_3F_3\div D$	-35.31×10^{-6}	-4.678×10^{-5}	$.675\times10^{-5}$	$-.612\times10^{-5}$

Amp. $=Y_3=82.6\times10^{-6}$. For phase angles see page 806, sec. 3.

Due to Machine No. 1

⑨ $n^2\omega_1{}^2$	3.55×10^2	14.21×10^2	32.0×10^2	56.9×10^2
⑩ $(\alpha_2+\alpha_3)$	$.46\times10^2$	$.46\times10^2$	$.46\times10^2$	$.46\times10^2$
⑪ $[⑩-⑨]$	-3.09×10^2	-13.75×10^2	-3.54×10^2	-56.44×10^2
$G_1=T_{s_1}[P_4/P_1]⑪$	-139.7×10^6	-681.7×10^6	-1426×10^6	-2552×10^6
$\psi_n=G_1F_1\div D$	-144.6×10^{-6}	-18.81×10^{-5}	-2709×10^{-5}	-2.460×10^{-5}

Amp. $=Y_1=337.2\times10^{-6}$. For phase angles see page 806, sec. 1.

$Y_0=Y_1+Y_2+Y_3+Y_4=17.380\times10^{-3}$.

Per cent. power pulsation $=Y_0\left(\frac{T_{s_4}\ S\times100}{7040\text{ k.w.}}\right)=58.8$ per cent.

impressed voltage varied over a considerable range, the maximum current will occur when the impressed frequency is

$$\omega = \sqrt{\frac{1}{LC}}$$

and the current flowing will be $\frac{E}{r}$.

This is called the *resonance* frequency of the circuit. It is that frequency for which the current due to an impressed voltage of a given magnitude is a maximum and its value is limited only by the resistance. It may be noted that in this simple circuit, the resonance frequency is independent of the resistance, while the natural frequency is not.

Comparison of Series Circuit and a Synchronous Machine.—The equation of the series circuit is

$$\left(r + Lp + \frac{1}{Cp}\right) i = e \qquad (1)$$

While the equation representing the movement of the rotor of a single synchronous machine, connected to an infinite system, has been shown to be:

$$(T_s + T_d p + I p^2) \psi = f(t) \qquad (2)$$

Equation (2) can be written to correspond with (1) as follows:

$$\left(T_d + Ip + \frac{T_s}{p}\right)(p\psi) = f(t) \qquad (2a)$$

Comparing equations (1) and (2a) it can be seen that

T_d	is	analogous	to	r
I	"	"	"	L
$\frac{1}{T_s}$	"	"	"	C
$p\psi$	"	"	"	i
$f(t)$	"	"	"	e.

Hence for the simple case of one machine connected to an infinite system the resonance frequency is, by analogy,

$$\omega = \sqrt{\frac{T_s}{I}} \qquad (3)$$

If resonance in mechanical systems is defined as that frequency for which the oscillation is theoretically limited only by the

damping torque T_d[10], the same result can be obtained from the expression for the oscillation of the rotor, which was shown to be:

$$\psi = \frac{Q_n}{I D_n} \sin\left(n\omega t + \tan^{-1}\frac{b_n}{a_n} - \tan^{-1}\frac{N_n}{M_n}\right).$$

where:

$$D_n = \sqrt{\left(\frac{T_s}{I} - \omega\right)^2 + \left(\frac{T_d}{I}\omega\right)^2}$$

Now, according to the definition, the condition for resonance is that ψ becomes theoretically infinity when $T_d = 0$. In other words, $D_n = 0$, when $T_d = 0$, is the condition for resonance. This gives:

$$\left(\frac{T_s}{I} - \omega^2\right) = 0, \text{ or } \omega = \sqrt{\frac{T_s}{I}} \text{ as before.} \qquad (3)$$

Resonance in a System of Two Synchronous Machines.—It is not so easy to extend the electrical analogy to systems of several machines, but it is a simple matter to calculate the resonance frequencies directly, if by resonance frequency is meant a frequency for which the oscillation is limited theoretically only by the damping.

In a system of two machines, the denominator of the expression for the oscillation was shown to be:

$$D_n = \sqrt{(D - B\omega^2)^2 + (C\omega - A\omega^3)}$$

The values of A, B, C and D are given in (7), page 615.

Putting the T_d's = 0 in this expression,

$$A = 0,\ C = 0$$

$$B = T_{s_2} + \left(\frac{P_2}{P_1}\right)^2 T_s,\ D = T_{s_1} T_{s_2}\left[\frac{1}{I_1} + \frac{1}{I_2}\left(\frac{P_2}{P_1}\right)^2\right].$$

So that for resonance,

$$\omega = \sqrt{\frac{D}{B}} = \sqrt{\frac{\frac{T_{s_1}}{I_1}\left[1 + \left(\frac{P_2}{P_1}\right)^2 \frac{I_1}{I_2}\right]}{\left[1 + \left(\frac{P_2}{P_1}\right)^2 \frac{T_{s_1}}{T_{s_2}}\right]}} \qquad (4)$$

[10] Theoretically it would be more correct to define a resonance frequency as that frequency for which the impressed harmonic produces a maximum oscillation. For a single machine connected to an infinite system, both definitions give the same value. For more complicated systems of two and three machines, the values of the resonance frequencies calculated by the first definition are not the same as those calculated by the latter, but the differences are so small that they can be neglected in all practical work. It would be very difficult to obtain mathematical expressions for the resonance frequencies in cases of more than two machines, using the latter definition of a resonance frequency.

This is the resonance frequency of two machines in parallel. If both machines are exactly alike, this formula shows that the resonance frequency of the combination is the same as that of one machine connected to an infinite system.

Resonance in a System of Three Machines.—The denominator of the expression for the oscillation in this case has been shown to be: [11]

$$D_n = \sqrt{(N_1\omega^4 - N_2\omega^2 + N_3)^2 + (M_1\omega^5 - M_2\omega^3 + M_3\omega)^2}.$$

Putting T_d's = 0,

$$D_n = N_1\omega^4 - N_2\omega^2 + N_3$$

so that for resonance,

$$\omega = \sqrt{\frac{N_2}{2N_1} + \sqrt{\left(\frac{N_2}{2N_1}\right)^2 - \frac{N_3}{N_1}}}$$

The values of N_1, N_2 and N_3 to be used in this formula are given in equation (7), page 789. It is interesting to note that in this case there are two roots and consequently two resonance frequencies.

Resonance in a System of "n" Machines.—It has been shown that for four machines the denominator of the expression for the oscillation, damping having been neglected, was

$$M_1 p^6 + M_2 p^4 + M_3 p^2 + M_4$$

where the M's are certain functions of α_n and β_n, given in equation (10), page 801. For n machines the denominator would be:

$$M_1 (p^2)^{n-1} + M_2 (p^2)^{n-2} + \cdots + M_{n-1} (p^2)^1 + M_n$$

where:

$$M_1 = \left[\beta_1 + \beta_2 + \cdots + \beta_n\right]$$

$$M_2 = \beta_1\left[\text{Combinations of } \alpha_n \text{ taken one at a time excepting terms containing } \alpha_1\right]$$
$$+ \beta_2\left[\text{Combinations of } \alpha_n \text{ taken one at a time excepting terms containing } \alpha_2\right]$$
$$\vdots$$
$$+ \beta_n\left[\text{Combinations of } \alpha_n \text{ taken one at a time excepting terms containing } \alpha_n\right]$$

[11] See page 789.

$$M_3 = \beta_1 \left[\begin{array}{l}\text{Combinations of } \alpha_n \text{ taken two at a time excepting terms} \\ \text{containing } \alpha_1\end{array}\right] + \beta_2 \left[\begin{array}{l}\text{Combinations of } \alpha_n \text{ taken two at a time excepting terms} \\ \text{containing } \alpha_2\end{array}\right]$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$+ \beta_n \left[\begin{array}{l}\text{Combinations of } \alpha_n \text{ taken two at a time excepting terms} \\ \text{containing } \alpha_n\end{array}\right]$$

The other M's are found in exactly the same way.

In the above expression:

$$\alpha_n = \frac{T_{sn}}{I_n} \text{ and } \beta_n = T_{sn}\left(\frac{P_1}{P_n}\right)^2$$

The resonance frequencies in a system of n machines are obviously the roots of the denominator expression. For n machines this expression is of the $(n-1)$st degree in p^2, so that there are theoretically $(n-1)$ resonance frequencies in a system of n machines.

Determination of the Resonance Frequencies in a System of Three Machines by the General Method.—The denominator of the expression for is:

$$M_1 (p^2)^2 + M_2 (p^2) + M_3 = 0$$

where:

$$M_1 = (\beta_1 + \beta_2 + \beta_3)$$
$$M_2 = \beta_1 (\alpha_2 + \alpha_3) + \beta_2 (\alpha_1 + \alpha_3) + \beta_3 (\alpha_1 + \alpha_2)$$
$$M_3 = \beta_1 (\alpha_2 \alpha_3) + \beta_2 (\alpha_1 \alpha_3) + \beta_3 (\alpha_1 \alpha_2)$$

in which:

$$\alpha_n = \frac{T_{sn}}{I_n}, \ \beta_n = \left(\frac{P_1}{P_n}\right)^2 T_{sn}$$

Putting $p^2 = -\omega^2$, the equation in p^2 becomes:

$$M_1 \omega^4 - M_2 \omega^2 + M_3 = 0$$

Solving for ω:

$$\omega = \sqrt{\frac{M_2}{2M_1} \pm \sqrt{\left(\frac{M_2}{2M_1}\right)^2 - \frac{M_3}{M_1}}}$$

which are the two resonance frequencies in a system of three synchronous machines.

Illustrative Examples.—Two illustrative examples will be worked out, one showing a typical calculation of the resonance

frequency in a combination of two machines, the other showing the calculation of the two resonance frequencies in the system of three machines for which the power pulsation was calculated on Plate III and found to be 442 per cent.

Resonance Frequency in Two Machines.—Item No. 1.

Engine.—3-cylinder, single acting, 4-cycle.
Flywheel $WR^2 = 45{,}000$.

Generator.—26 pole—125 k.w.—277 R.P.M. $P_o = 430$.
Rotor $WR^2 = 4180$.

Total $WR^2 = 49{,}180$.

$$I_1 = 1.527 \times 10^3, \quad T_{s_1} = 3520 \left(\frac{430 \times 26}{277}\right) = 1.42 \times 10^5 \text{ [12]}$$

$$\therefore \quad \alpha_1 = \frac{T_{s_1}}{I_1} = 93, \qquad \beta_1 = 1.42 \times 10^5.$$

The resonance frequency of this one machine on an infinite system is

$$\omega = \sqrt{\frac{T_{s_1}}{I_1}} = \sqrt{\alpha_1} = 9.65 \text{ radians per second.}$$

The lowest harmonic based on half the revolutions has a frequency of

$$\frac{277}{2} \text{ R.P.M. or } 14.5 \text{ radians per second.}$$

Now 9.65 is 33.4 per cent. below the frequency of the lowest impressed force which is 14.5. Since this is a three-cylinder engine, this would certainly be a conservative value and this machine would parallel perfectly with an infinite system.

Item No. 2.

Engine.—4-cylinder, single acting, 4-cycle.
Flywheel $WR^2 = 15{,}000$.

Generator.—175 k.w.—277 R.P.M.—26 pole. $P_o = 590$.
Rotor $WR^2 = 6000$.

Total $WR^2 = 21{,}000$. $I_2 = 652.5$.

$$T_{s_2} = 3520 \left(\frac{590 \times 26}{277}\right) = 1.948 \times 10^5 \text{ [12]}$$

$$\alpha_2 = \frac{T_{s_2}}{I_2} = 298, \quad \beta_2 = 1.948 \times 10^5$$

[12] See formula for T_s, page 607.

The resonance frequency of this one machine on an infinite system is:

$$\omega = \sqrt{\frac{T_{s_2}}{I_2}} = \sqrt{\alpha_2} = 17.3$$

which is 19.3 per cent. above 14.5, the frequency of the lowest harmonic based on half the revolutions. This machine would parallel satisfactorily with an infinite system.

When the units are connected in parallel the resonance frequency of the combination is:

$$\omega = \sqrt{\frac{\frac{T_{s_1}}{I_1}\left[1 + \left(\frac{P_2}{P_1}\right)^2 \frac{I_2}{I_1}\right]}{\left[1 + \left(\frac{P_2}{P_1}\right)^2 \frac{T_{s_1}}{T_{s_2}}\right]}} = 13.4 \text{ [13]}$$

This value is only 7.6 per cent. below 14.5, the frequency of the lowest harmonic based on half the revolutions. This is too near for safety. If there were no unbalancing in the power delivered by the different cylinders, there would be no harmonic based on half the revolutions. However, if it is assumed that one cylinder of each engine gives 20 per cent. less power than the others, satisfactory parallel operation would not be obtained.

Here then is a case, where two machines would give satisfactory operation by themselves on an infinite system but could not be paralleled with each other safely. In this particular case the difficulty was overcome by using a larger flywheel on the four-cylinder engine so that the resonance frequency of the combination was about 25 per cent. below the frequency of the lowest harmonic.

Two Resonance Frequencies in a System of Three Machines.—The data for these machines are given on page 792. From these data it is found that:

$$\alpha_1 = 1.57 \times 10^2 \qquad \beta_1 = 8.67 \times 10^5$$
$$\alpha_2 = 1.57 \times 10^2 \qquad \beta_2 = 8.67 \times 10^5$$
$$\alpha_3 = .393 \times 10^2 \qquad \beta_3 = 12.1 \times 10^5$$

The individual resonance frequencies, that is, the resonance frequencies of the individual machines when operated on an infinite system, are:

$$\sqrt{\alpha_1} = \sqrt{\alpha_2} = 12.55$$
$$\text{and } \sqrt{\alpha_3} = 6.28$$

[13] See page 814.

The resonance frequencies in the combination of the three machines in parallel can be found by the general method given on page 814 as follows:

$$M_1 = \beta_1 + \beta_2 + \beta_3 = 29.44 \times 10^5$$
$$M_2 = \beta_1(\alpha_2 + \alpha_3) + \beta_2(\alpha_1 + \alpha_3) + \beta_3(\alpha_1 + \alpha_2) = 72.0 \times 10^7$$
$$M_3 = \beta_1(\alpha_2\alpha_3) + \beta_2(\alpha_1\alpha_3) + \beta_3(\alpha_1\alpha_2) = 40.55 \times 10^9$$

$$\omega = \sqrt{\left(\frac{M_2}{2M_1}\right) \pm \sqrt{\left(\frac{M_2}{2M_1}\right)^2 - \frac{M_3}{M_1}}} \text{ from which}$$

$$\omega_1 = 12.48 \text{ and } \omega_2 = 9.42.$$

It is interesting to compare these two resonance frequencies of the combination with the three individual resonance frequencies, which were 12.55, 12.55 and 6.28. It should be particularly noted that the resonance frequencies of the combination fall within the range of frequencies comprehended between the greatest and the smallest of the individual resonance frequencies. This seems to be a perfectly general property which holds for any number of machines. No doubt, it could be proved mathematically as a general theorem.

Now, in the above three machines, the frequencies of the lowest impressed harmonics corresponding to half the revolutions are:

7.85 and 10.45

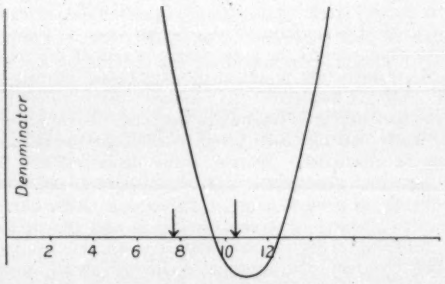
one of which is within the range of the resonance frequencies 9.42 to 12.48 and the other slightly below it. If the denominator of the expression for power pulsation were plotted as a function of the frequency of the impressed force it would resemble the graph shown in Fig. 6. Resonance occurs at the points where the curve crosses the axis and the positions of the arrows indicate the frequencies of the impressed forces. It can be seen that the ordinates corresponding to the positions indicated by the arrows are relatively small, especially for the arrow at the value 10.45. It is not surprising, therefore, that the expressions for power pulsation show extremely high values,[14] the maximum being 442 per cent.

But how, in this case, is the difficulty to be remedied? Obviously, the flywheels must be changed so as to bring the resonance frequencies both below the frequency of the lowest im-

[14] See page 799.

pressed harmonic, which is 7.85, due to the four-cylinder engine. There is no possibility of bringing the resonance frequencies between the first and second harmonics, for there is not room enough between 10.45 and 15.7, the frequency of the second harmonic (corresponding to the revolutions) due to the four-cylinder engine. To bring the resonance frequencies down to 20 or 25 per cent. below 7.85 would require four or five times as large flywheels on the six-cylinder engines. It is obvious that

FIG. 6.



the manufacturer made a very unfortunate choice when he decided to parallel these two six-cylinder engines with a four-cylinder of much slower speed. If he had used three six-cylinder engines of the same type, no doubt perfect parallel operation would have been secured with the resonance frequencies between the first and second impressed harmonics. Possibly better results could have been secured with even lighter flywheels than those already on the six-cylinder engines.

In conclusion, the writer wishes to call particular attention to the importance of a study of the resonance frequencies in combinations of engine-driven generators, which are to parallel successfully. By the general methods developed in this paper it takes but a few minutes to make a complete study of any particular combination. It will show that in some cases light flywheels will give successful operation while in other cases heavy ones may be required. Unfortunate combinations of machines, such as the one just discussed, can be recognized immediately.

Extensive or elaborate calculations of the power pulsation are not necessary to enable a manufacturer to select correct flywheels for his engines. But in all cases where a knowledge of the resonance frequencies gives indication that trouble is likely to occur, the matter can be settled definitely by resorting to an actual calculation of the power pulsation by the methods outlined in this paper.

Gaseous Combustion at High Pressures.—Doctor Bone contributed lately to the Royal Institution, results of studies of combustion at very high pressures. The scientific investigation of flame as a distinct phenomenon was begun over a century ago, by Davy, whose invention of the safety-lamp is one of the most striking instances of an important practical result obtained by careful experimentation. Davy's initiative has, indeed, been influential among chemists for the subject is constantly pursued. Notwithstanding the amount of study that has been given to combustion-chemistry, much remains to be elucidated. Doctor Bone describes in considerable detail the apparatus used, but it will be sufficient to say that specially designed vessels of forged steel were used to withstand the high initial pressure running in some cases up to 100 atmospheres. The pressures resulting from the combustion may run up to as much as ten times as great, the increase occurring almost instantly. It is recorded in the report that one of the assistants was killed in consequence of an accident in the course of the experiments. Pressures produced during explosion were recorded on films. Hydrogen and carbon dioxide were the combustibles used. These gases produce under similar conditions almost the same amount of heat, but the flames are very different in appearance and in several properties. It was shown long ago that combustion of carbon monoxide depends on the presence of a small amount of water vapor. This fact was shown experimentally by Doctor Bone by turning a jet of burning carbon monoxide into a vessel containing perfectly dry air, when the flame was extinguished. Flame is transmitted much more slowly through a mixture of carbon monoxide and air than through a mixture of hydrogen and air. Incidental to these studies of high pressure combustions, Bone found that nitrogen can be "activated," that is, its combining activity increased by the combustion of carbon monoxide with oxygen at high pressures when hydrogen is as far as possible excluded. This fact may lead to a method of fixing nitrogen, a problem that is now of great importance. H. L.

THE RELATION OF SOUND VELOCITY TO HEIGHT.*

BY

W. J. HUMPHREYS, C.E., Ph.D.

Meteorological Physicist, U. S. Weather Bureau.
Associate Editor.

THE elastic wave in the atmosphere caused by the explosion of Krakatoa on August 27, 1883, travelled, as is well known, several times around the earth at the average velocity of about 315 metres per second. That is, at a velocity that averaged about sixteen metres per second less than that of sound in air at normal pressure and 0° C. temperature. Other long-distance air waves also have seemed to travel comparatively slowly. Presumably, therefore, this phenomenon is owing to some persistent and ubiquitous cause.

The effects of wind on the velocity of "sound," whether audible or inaudible, are well known, and were more or less averaged out in the Krakatoa and other similar observations. Hence, provisionally, we may regard the phenomenon in question as pertaining to still air and the velocity as being relative to a fixed point on the surface of the earth. Conceivably, then, the comparatively slow velocity of far-off "sounds" may be apparent only and owing to a more or less zigzag path in a vertical plane, resulting from reflections and refractions. Or, perhaps, to travelling part of the way, at least, through the upper air, provided the velocity of sound is decidedly less at considerable heights than at the surface of the earth.

To test the adequacy of these causes (the second, though necessarily implying also some of the first, seems to be of the right order), and for whatever other purpose they may serve, I have computed the average winter and summer velocities of sound at various altitudes from sea-level to 16 kilometres, as per the closely known values of density and pressure in the free air over northwestern Europe.[1] These velocities would, of course, in every case, be directly proportional to the square root of the absolute temperature at the place under consideration if the

* Communicated by the Author.

[1] W. J. Humphreys, "Physics of the Air," regular edition, p. 72.

composition of the air were constant. But this composition is not constant, owing, almost entirely, so far as the troposphere is concerned, to changes in humidity. Hence the velocities given in Table I, and represented graphically in Fig. 1, have been computed, as stated, from average densities and pressures; and also

TABLE I.

Height Km.	Summer Velocity. m/s	Winter Velocity. m/s
0.0	340.72	332.56
0.5	339.98	332.25
1.0	338.86	331.09
1.5	337.12	330.07
2.0	335.41	328.93
2.5	333.76	327.53
3.0	332.12	325.72
4.0	328.81	321.90
5.0	325.26	317.45
6.0	321.26	312.93
7.0	316.81	308.21
8.0	312.00	303.61
9.0	307.14	299.62
10.0	302.79	296.44
11.0	299.32	294.63
12.0	298.02	294.63
13.0	298.02	294.63
14.0	298.02	294.63
15.0	298.02	294.63
16.0	298.02	294.63

further corrected slightly for the effect of the water vapor present on the ratio of the specific heats.

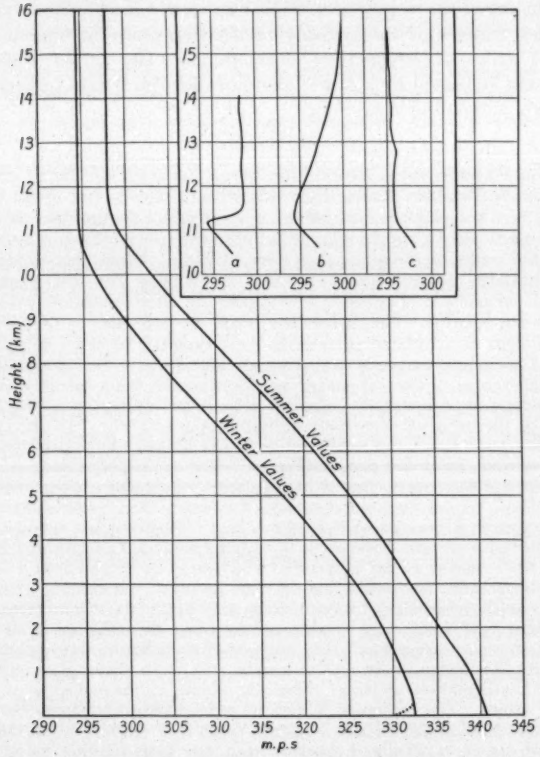
In general, the velocity of sound in the atmosphere decreases four metres per second, roughly, per kilometre increase of height throughout the troposphere, and is nearly constant, through the given range of level, within the stratosphere. Here (in the stratosphere) both the temperature and the composition, up to 16 kilometres, are practically constant, hence the constancy at these levels of sound velocity.

During the latter portions, especially, of still, clear winter nights, that is, whenever there is a marked temperature inversion, the velocity of sound increases with altitude through the first 100 or 200 metres, as indicated by the dotted branch of the curve of winter values. Hence a sound originating within this inversion

layer is largely confined thereto, by alternate reflection from below and refraction, simulating reflection, from above.

Since the height and temperature of the top of the troposphere

FIG. 1.



Average velocity of sound at different heights.

both vary more or less from day to day, it follows that the *average values* of the velocities of sound may, and actually they do, gradually change on approach to and passage into the strato-

sphere, while the *actual values* change more or less abruptly and irregularly. The curves of the main figure show this gradual change of the average values of the velocity of sound with height through the range (roughly from the 10 to the 12 kilometre level) of the top of the troposphere. The inserts, *a*, *b*, and *c*, show typical actual changes of this velocity as determined by inversions and other types of temperature distribution. The rather common changes indicated by *a* and *b* obviously lead to sound refractions closely simulating reflections.

An Experimental Determination of the Rate of Decay of the Short-life Product Radium C'. J. C. JACOBSEN. (*Phil. Mag.*, Jan., 1924.)—This product has indeed a short life. Geiger and Nuttall have established a simple relation between the transformation constant and the range of the alpha rays emitted in the change that holds for the uranium-radium family. Since C' in changing into RaD gives off alpha particles, it is possible to compute its transformation constant from the relation. This comes out equal to 5.10^7 sec.$^{-1}$. From this value there is calculated further that of a quantity of RaC' in $.35.10^{-7}$ sec. half is automatically transformed into RaD. The Copenhagen physicist has, it thus appears, set for himself the modest task of measuring experimentally, whether directly or indirectly, a length of time of the order of one-thirty-millionth of a second. "If one could prepare pure RaC', the substance would practically have disappeared 10^{-6} seconds after the preparation."

Here is the foundation of his method: "Consider a plate covered with RaC. From the plate a stream of atoms of RaC' will be sent out moving with a velocity of $8.4.10^5$ cm./sec. When such a recoil atom has moved a certain distance, it is transformed into an atom of RaD with the emission of an α-particle, which may be detected on a zinc-sulphide screen. By measuring the diminution of the number of RaC' atoms with increasing distance from the plate, the transformation constant λ of RaC' may be determined from the velocity V of the recoil atoms or *vice versa*." The number of scintillations were counted at different distances from the source and from these observations the transformation constant came out equal to the velocity of the RaC' atoms. The difficulty is that no very trustworthy data on the velocity are known. There is some reason to hold it to be about $8.4.10^5$ cm./sec., as above quoted. On the basis of this value the transformation constant also equals the same number. Since this value is less than that assumed at the beginning, it follows that a longer time passes before RaC' is half transformed into RaD. This time, according to the new results, has lengthened into one five-hundred-thousandth second. G. F. S.

THE ROTATION OF MELTING ICE SUSPENDED IN BENZINE.*

BY

H. B. HACHEY, B.Sc.

A PAPER written by A. Artom concerning the formation of hail was published on June 18, 1922, in the *Accad. Lincei. Atti* (31 i, pp. 513–518). In discussing this paper, *Science Abstracts* (No. 306, June 25, 1923) makes the following remarks: "The electrical conditions for the formation of hail are discussed. This takes place in a medium rendered feebly conducting by ionization in which occur insulating bodies, such as the nuclei of the hail. The ionization is due (i) to the emissions from the sun regarded as an incandescent body at a high temperature sending out corpuscles negatively electrified, and (ii) to the action of the ultra-violet rays. Hail is usually seen in the hottest part of the day when the ionization by ultra-violet rays is strongest. Hail is formed frequently between rain clouds with high electric charges, and water vapor in the presence of ice has the property of feeble electric conductivity. In these circumstances the electric field causes an action of rotation which acts on the small drops of water. A piece of ice suspended in benzol begins to rotate and continues this motion as long as the suspending thread permits. Such a rotating mass forms a solid of revolution and such are seen in the case of hail. Helicoidal forms due to the additional action of gravity are also observed."

It is quite improbable that such a theory of the formation of hail could replace the precise explanation of Simpson, which is generally accepted at the present time. However, the writer's statement that a piece of ice suspended in benzol begins to rotate and continues this motion as long as the suspending thread permits, led to an investigation to determine:

(i) Whether ice does rotate when suspended in benzol;

(ii) Whether this rotation is a surface effect or a volume effect;

(iii) Whether it was true of liquids other than benzol.

* Communicated by Dr. A. S. Eve, Director of Physics, McGill University, and Associate Editor of this JOURNAL.

It was first necessary to obtain a suspension thread that would hold the portion of ice to be suspended and yet offer very little resistance to a slight turning moment. Again it was necessary to make certain that all twist was taken out of the suspension thread so that the results obtained would be due to the effects of ice in benzol.

Silk fibre was chosen as the suspension thread, and by attaching a weight (approximately equal to that of the ice) and leaving it in suspension over night, all twist in the thread was taken out. The ice was suspended in benzol whose temperature was about 20° C., and it was seen that the ice revolved until the suspension thread stopped the motion. This was repeated with the benzol at temperatures ranging from 20° C. to −10° C. and it was seen that the speed of revolution decreased with the lowering of the temperature until it reached the point (somewhere between 5° C. and 0° C.) where all motion ceased or was practically negligible. When a temperature of −10° was used, there was absolutely no motion of revolution noticeable. Then the height of the ice from the bottom of the vessel was varied and it was seen that the speed of revolution varied with the height of the ice above the bottom of the vessel, *i.e.*, the greater the height the greater was the speed of revolution, always, of course, provided that a portion of the ice was under the surface. Again, the speed depended to a certain extent upon the amount of ice immersed, for the revolving motion was much more rapid when the ice was completely immersed than when it was only partially immersed (keeping the height of the ice above the bottom of the vessel constant). It was also noticed that the direction of rotation was not constant and that when the same piece of ice was inverted it did not always turn in the opposite direction.

All through this investigation it was apparent that when the ice had motion of revolution (when the temperature of benzol was above 0° C.) a steady stream of water flowed from the ice to the bottom of the vessel containing the benzol. As soon as large drops, which also formed on the lower end of the ice, gave way and were precipitated to the bottom of the vessel, the rotation of the ice was accelerated.

The experiment was repeated with water and also with gasoline. When ice was suspended in water there was absolutely no motion of revolution noticeable; but when the ice was suspended in

gasoline the results obtained were similar to those obtained with benzol.

From the results obtained the natural conclusion is:

(i) That ice when suspended in benzol does rotate.

(ii) That the speed of rotation decreases as the temperature of the benzol is lowered and all rotation ceases when the benzol is at the temperature of 0° C.

(iii) The rotation also takes place when gasoline is used, but in the case of water the ice did not rotate.

(iv) That the rotation is probably due to the downward flow of the denser melted water producing a torque owing to its irregular motion reacting on the ice, particularly when the surrounding medium is much less dense.

Use of Diphenylamine as an Indicator in the Volumetric Determination of Iron.—J. KNOP (*Jour. Am. Chem. Soc.*, 1924, 46, 263–269) recommends the use of diphenylamine sulphate as an internal indicator for the titration of ferrous salts with standard potassium dichromate solution. Three drops of a 1 per cent. solution of diphenylamine in concentrated sulphuric acid are used as an indicator in each titration. The end-point occurs when the addition of one drop of the potassium dichromate solution produces an intense violet-blue color, unchanged on further addition of that reagent. The method may be applied to ferric salts, which have been reduced by stannous chloride followed by addition of mercuric chloride. This method also permits back-titration. J. S. H.

Mercury as an Industrial Poison.—J. A. TURNER, of the United States Public Health Service (*Pub. Health Rep.*, 1924, 39, 329–341), has studied several cases of mercurial poisoning, due to the passage of mercury vapor into the atmosphere from the mercury gaps in induction furnaces. He finds that the signs and symptoms of mercurial poisoning are produced in man by exposure, for several hours daily during a period of two or three months, to an atmosphere containing as small a quantity as 0.02 milligram of mercury per cubic foot of air. If the daily period of exposure be between three and five hours, the daily absorption of mercury ranges between 0.771 and 1.285 milligrams. The symptoms develop chiefly in the mouth and gastro-intestinal tract; the blood and urine are not affected. The mercury vapor from the furnaces condenses, for dust from the furnace room contained from 1 to 3 per cent. of metallic mercury. Poisoning of this nature is best prevented by inclosing all apparatus in which mercury is used, and by conveying the fumes away from the worker's face so that it will be impossible for him to inhale them. J. S. H.

A Determination of the Vapor Pressures of Cæsium and Rubidium. D. H. SCOTT. (*Phil. Mag.*, Jan., 1924.)—The method used was that published by Haber in 1914. A quartz fibre .7 or .8 mm. in diameter and some 8 cm. long, supported in a wide glass tube, is illuminated from the side and observed through a microscope. "When the fibre is at rest a sharp line of light is seen in the field of view, and if the fibre is set in vibration this line broadens out into a band whose width can be measured on a scale in the eyepiece of the microscope. The method of using the instrument consists in observing the time taken for the amplitude of vibration to diminish to a definite fraction of its original value." Let there be more than one kind of gas present in the space surrounding the fibre as it executes vibrations in a plane. For each of these gases form the product of its pressure multiplied by the square root of its molecular weight. Add the separate products together and to the sum add a quantity *a*, a constant. Then the product of this final sum by the number of seconds required for the bright band to diminish to a definite fraction of its original width equals another constant, *b*. The values of *a* and *b* are obtained by measuring the pressures of known gases by means of a McLeod gauge in connection with observations of the times of dying down. Once these constants have been determined the relation given above may be used to get the pressure of a gas whose molecular weight is known.

As a test the vapor pressure of mercury at 20° C. was found to equal $1.30.10^{-3}$ mm. This is an intermediate value in comparison with those got by others. When used for cæsium and rubidium the apparatus had to be kept in an air-bath at 150° C. to avoid condensation of the metals. The metal was contained in a tube connected to the main tube in which the fibre vibrated, where it was electrically heated. The pressures were determined for temperatures ranging approximately from 50° to 130° C. Formulas for the pressures are given. By using the letter *T* to indicate two different things, the author has added unnecessary difficulty to his paper. G. F. S.

Hydrates of Lime.—R. T. HASLAM, G. CALINGAERT, and C. M. TAYLOR, of the Massachusetts Institute of Technology (*Jour. Am. Chem. Soc.*, 1924, **46**, 308–311), have demonstrated that lime forms only the monohydrate $CaO.H_2O$ or $Ca(OH)_2$. Efforts were made to prepare a dihydrate by evaporation of lime water at a temperature of 30° C. in dry air and in a vacuum, by precipitation of solutions of calcium salts by means of potassium hydroxide, and by long contact of hydrated lime with water at a temperature of 55° C. All these procedures yielded only the monohydrate. Redetermination of the solubility curve for calcium oxide in water showed definitely that no dihydrate exists. J. S. H.

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

A CAMERA FOR STUDYING PROJECTILES IN FLIGHT.[1]

By H. L. Curtis, W. H. Wadleigh and A. H. Sellman.

[ABSTRACT.]

WITH very rapidly moving objects it is not possible to take a clear picture on a stationary film, since in the interval during which the film is exposed, the image of the object moves sufficiently to blur the picture. The present camera overcomes this difficulty by causing the film to move with approximately the same velocity as does the image of the object. Hence there is no blurring of the picture.

In order to secure a number of pictures, several lenses are placed at right angles to the direction of motion of the film and a focal plane shutter so arranged that the images from these lenses are allowed to fall on the film in succession. This gives a series of pictures of a stationary object which lie diagonally across the film. However, if the velocity of the image is the same as the velocity of the film, the pictures of the moving object will lie across the film perpendicular to the direction of motion.

The velocity of the object can be determined from the velocity of the image if the magnification of the lenses is known. This can be determined from the focal length of the lenses and the distance of the object from the camera. By measuring the angle between a line perpendicular to the direction of motion of the film and a line joining successive pictures of the moving object, it is possible to determine the velocity of the image from the velocity of the film. The velocity of the film can be accurately determined by throwing on it flashes of light from a vibrating tuning fork. By this method, the velocity of an object can be obtained with an accuracy of at least 1 per cent.

The camera consists of a drum for carrying the film, a focal plane shutter drum, several lenses arranged in a line parallel to the axis of the drums, a motor for driving the drums, a tuning fork for timing the film, and a magnetically operated shutter. The

* Communicated by the Director.

[1] Technologic Papers, No. 255, price ten cents.

focal plane shutter drum has slots so arranged that exposures are made by the different lenses in succession. The shutter drum rotates in the opposite direction from the film drum, and with a speed several times as great. The lenses should all have the same focal length and be placed at the same distance from the film.

The camera has been used to take pictures of projectiles in flight. With five lenses, 250 pictures per second have been taken. By increasing the number of lenses, the number of pictures can be increased. It should be useful whenever it is desired to study objects which are moving with high velocity.

RADIO INSTRUMENTS AND MEASUREMENTS.[2]

[ABSTRACT.]

IN the rapid growth of radio communication, the appliances and methods used have undergone frequent and radical changes. In this growth, progress has been made largely by new inventions and applications, and comparatively little attention paid to refinements of measurement. In consequence, the methods and instruments of measurement peculiar to radio science have developed slowly and have not yet been carried to a point where they are as accurate or as well standardized as some other electrical measurements.

This circular presents information regarding the more important instruments and measurements actually used in radio work. The treatment is of interest to Government officers, radio engineers, and others. Many of the matters dealt with are or have been under investigation in the laboratories of the Bureau of Standards and are not treated in previously existing publications. No attempt is made in this circular to deal with the operation of apparatus in sending and receiving. The bureau's publications on that and other radio subjects are listed in Appendix 2.

The first edition was issued March 23, 1918. A number of corrections and revisions have been made in the present edition. The bibliography of radio publications has been considerably extended.

The first part of the circular gives a brief treatment of the principles of radio-frequency currents, upon which all measurements are based. This treatment presents many simplifications

[2] Circular No. 74, second edition, price sixty cents.

of radio theory. This part of the circular makes it serve as a basic treatise or an introduction to other technical radio publications. Some of the subjects included under principles are resonance, coupled circuits, antennas, and damped currents.

Information on standards and methods of measurement is grouped under wave-meters, condensers, inductance coils, current measurement, and resistance measurement. Special attention is given to electron tubes, which are extensively used in radio measurements as generators of current, as detectors, and in other ways.

A large collection of formulas is given for radio calculations. These include calculation of capacity and inductance of antennas, coils, etc., design of inductance coils, data on radio-frequency resistance, and formulas for calculation of frequency, decrement, and many other radio calculations.

SUMMARY OF TECHNICAL METHODS FOR THE UTILIZATION OF MOLASSES, COLLATED FROM PATENT LITERATURE FOR THE USE OF THE AMERICAN SUGAR INDUSTRY.[1]

[ABSTRACT.]

IT has long been recognized that in beet molasses and the waste-water from saccharate processes of the beet-sugar industry there are valuable substances which should be utilized to the profit of the manufacturer. Attempts have been made since the middle of the last century to accomplish this end, but with little success, except in Germany, where it is known that plants have been erected for obtaining various by-products from these waste liquors. It has been possible to obtain very few details of the processes used in these plants.

In order to carry out experimental work for the American beet-sugar industry, the patents relating to the utilization of the non-sugars in beet molasses have been collected from the files of the patents of the United States, Great Britain, France and Germany as found at the U. S. Patent Office in Washington. It was assumed that, whether molasses was treated for the recovery of further sugar by a known saccharate process or used as a source of alcohol and yeast, there remained a waste-water or distillery slop from which the non-sugars might be economically recovered.

Through the work of various investigators, chiefly in France

[1] Circular No. 145, price fifteen cents.

and Germany, the nature of the various compounds to be found in beet-sugar waste-waters has been made known. Dahlberg [4] has summarized the possible products from beet molasses, exclusive of the sugar, which may be almost entirely removed by means of saccharate processes, as the following: Methylamines, ammonia or ammonium sulphate, sodium or potassium cyanides, nitrogen bodies, such as betaine and glutaminic acid, alkali salts—K_2CO_3, Na_2CO_3, K_2SO_4—vegetable carbon and tar, and various minor products, such as methyl alcohol, glycerine, organic acids and esters. These latter have received considerable attention in the patent literature because of their production from distillery slops or vinasses. A summary of the patents follows:

Group 1.—Amines.—Processes for producing methylamines from waste-water. The patents indicate that little attention has been given to the preparation of amines as such from waste-water or molasses. Methods for the preparation from other sources and the separation of the three methylamines are of interest from the research point of view.

Group 2.—(*a*) Production of ammonia directly from waste-water, discard molasses, or other sugar wastes. (*b*) Production of ammonia from other industrial wastes. (*c*) Separation of the ammonia from the products of processes of distillation or incineration. Means of absorbing, fixing or purifying ammonia are so generally known in connection with the by-product coal industry that it was deemed inadvisable to make any attempt to cover this angle of the process completely. (*d*) Further processes for the production of ammonium salts from the constituents of the gases of dry distillation.

The principles used in production of ammonia from waste-water or molasses are included in a few well-defined methods:

(1) Heating the waste-water with an excess of alkali in any suitable container, said alkali sometimes being obtained from the waste-water ash. (2) Passing the gases from the dry distillation of waste-water in contact with heated basic materials, such as lime, clays, bauxite, aluminates or mixtures of waste-water ash and aluminates, which are subsequently treated for the recovery of the alkalies, alumina, etc. Various methods of using briquetted aluminates as a contact mass are described. (3) Calcining the waste-water in contact with coke and steam to increase the yield of

[4] *Chem. Met. Eng.*, 23, 421.

ammonia. (4) Ammonium salts are obtained directly by heating with acids either with or without treatment with ferments.

Methods for obtaining ammonia from other industrial wastes are very similar to those noted above.

Under special conditions it is possible to prepare ammonium carbonate from the ammonia and carbon dioxide produced in the gases from the dry distillation of waste-water.

Group 3.—Apparatus.—Types of ovens, retorts and miscellaneous equipment for all groups are described. (*a*) Apparatus for incinerating and calcining waste-water. (*b*) Apparatus for evaporating and concentrating waste-water. (*c*) Apparatus for recovering ammonia from waste-waters. (*d*) Apparatus for the recovery of potash.

The development of retorts for dry distillation has been in the general direction of either rotating drums with stationary scrapers, or stationary retorts containing screw conveyors or other means of agitating and moving the incinerating mass. Many suggested pieces of equipment are highly complicated and could not be adequately reproduced, aside from the consideration of their practical utility.

Group 4.—Cyanides.—In this group it was found necessary to limit the number of related patents to a small fraction of those actually gathered from the patent literature. (*a*) Production of cyanides from waste-water direct. (*b*) Production of cyanides from amines by special processes. (*c*) Production of cyanides from various gas mixtures. (*d*) Recovery of hydrocyanic acid from gas mixtures. (*e*) Preparation of cyanides in marketable form.

The present methods for the preparation of cyanides from waste-water are developments of the original process of Ortlieb and Mueller [5] in the use of some type of superheater through which the gases from the dry distillation are passed either directly or after the separation of the cyanizable and non-cyanizable constituents. Reichardt and Bueb patented the first application of this principle, using a checkerwork of fire-brick heated to a red or white heat as a cyanizing chamber. Subsequent patents have changed the form of the superheater to tubes made of quartz, zirconia and other acidic materials.

[5] Ger. 9409. Actien Gesellschaft Croix.

One or all of the amines are also used in other ways than that described by Ortlieb and Mueller for producing cyanides, as in the process for passing the vaporized amine into molten sodium to produce the alkali cyanide directly.

By the use of catalysts, certain of the gases produced in the dry distillation of waste-water may be made to combine to form hydrocyanic acid.

Group 5.—Nitrogenous Non-sugars.—Patents not previously listed under amines, ammonia or cyanides, applying to nitrogenous compounds such as betaine and glutaminic acid are collected under the following groups: (*a*) Betaine and glutaminic acid from waste-water and molasses. (*b*) Betaine prepared from other sources.

The methods for preparing betaine and glutaminic acid from waste-water and molasses are in general variations of the treatment of those materials with either gaseous or aqueous hydrochloric acid and alcohol.

Group 6.—Potash and Alkali Salts.—Only those patents bearing directly on the waste-water problem have been taken for this survey. The recovery of potash and alkali salts from the waste-water ash is accomplished by two general methods: (1) Evaporating, igniting or calcining the waste-water and leaching the resulting product in various ways to separate the potassium and sodium compounds. (2) Precipitation of the potash either with hydrofluosilicic acid or perchlorates.

Another form of utilization, developed to a considerable extent in America during the late war, is found in the direct use of the dried ash as a fertilizer.

Group 7.—Recovery of Miscellaneous Products from Waste-water, Molasses, Distillery Slop, etc.—(*a*) Glycerine, fatty acids, esters, etc. (*b*) Fertilizers, feed, etc. (*c*) Dyeing and mordanting materials. (*d*) Miscellaneous products—alkalies, phosphates, etc.

Glycerine and fatty acids are obtained from waste-water by treatment of dried residue with solvents such as alcohol and acetone, or by fractional distillation under partial vacuum. The various specifications show slight changes from this procedure.

EMISSIVE TESTS OF PAINTS FOR DECREASING OR INCREASING HEAT RADIATION.[6]

By W. W. Coblentz and C. W. Hughes.

[ABSTRACT.]

THE object of this paper is to describe experiments conducted in quest of a means for decreasing or increasing the rate of thermal radiation from heated surfaces.

Data are given on the emissivity of sheet iron, cotton duck, roofing material, artificial leather, etc., covered with white paint, vitreous enamel, aluminum paint, etc.

It was found that aluminum paint emits only 30 to 50 per cent. as much thermal radiation as the unpainted material and as the white paint, vitreous glass enamel or other non-metallic coatings.

The data are useful in giving a means for reducing the heat radiated from the under side of roofs, tents, awnings, automobile tops, etc.

A coating of aluminum paint applied to the top of an opaque canopy, *e.g.*, automobile top, reduces by 50 per cent. the heat radiation from the under side. A coat of aluminum paint applied to the under side of a cotton duck tent cloth shuts out 85 per cent. of the heat rays.

The application of these data to house radiators is discussed. It is shown that, owing to the fact that house radiators are essentially convectors of heat, a gain of only 15 to 20 per cent. in heat dissipation into the room may be expected by covering the surface of the radiator with a paint which is free from flakes of metal, *e.g.*, aluminum or bronze.

REPORT OF THE SIXTEENTH ANNUAL CONFERENCE ON WEIGHTS AND MEASURES.[7]

[ABSTRACT.]

THE Annual Conference on Weights and Measures is composed of weights and measures officials, both state and local, from all parts of the country. Manufacturers and others interested in the subject are invited to attend the conferences.

The present report contains addresses by the Secretary of Commerce, the president of the conference, and the director of the Bureau of Standards; papers and discussions on such subjects

[6] Technologic Papers, No. 254, price five cents.

[7] Miscellaneous Publication No. 55, price thirty cents.

as sale of service on the basis of weight or measure, coöperation between departments, the organization and conduct of departments of weights and measures, commercial filling of milk and cream bottles and uniform regulations and proposed methods for testing such bottles, temperature as a factor in measurement of gasoline, problems arising in the supervision of public markets, the elimination of sales by gross weight, manner of sale of cordage, the abolition of bushel weights, standardization and simplification, bread weight legislation and labelling requirements for bread, origin and destination weighing of coal in carload lots, uniform ton for coal, and retail sale of coal and coke. Reports were made on bread weight legislation and tolerances, specifications and tolerances for fabric-measuring devices and for vehicle tanks used in the sale of gasoline, and tolerances for heavy-duty automatic scales.

SOME METHODS OF TESTING RADIO RECEIVING SETS.[1]

By J. L. Preston and L. C. F. Horle.

[ABSTRACT.]

THIS paper describes some methods of measurement of electrical characteristics of a radio receiving set and formulates statements of features which may be learned by an inspection of the electrical and mechanical design of a set. In developing these methods it has been the aim to provide means for determining to what extent a receiving set embodies the following characteristics:

(1) Sufficient sensitivity to produce audible or loud sounds when tuned to receive from stations which may be located at a considerable distance. Sensitivity may be defined as the ratio of the telephone-receiver signal current to the voltage impressed on the antenna circuit of the receiving set, or sensitivity $= I_t/E_a$. When I_t is expressed in amperes and E_a in volts, it has been found more convenient to multiply the quotient by some factor, such as 10_5, in order to avoid values of sensitivity that are much less than unity. It was found essential, when comparing various types of receiving sets, to determine the sensitivity at several impressed voltages, ranging from about 1 to 100 mv.

(2) Selectivity, or the ability to respond to signals of a given frequency without responding to signals of slightly different frequency. Selectivity is expressed in terms of a quantity called the

[1] Technologic Papers, No. 256, price ten cents.

sharpness of resonance, which is computed by some form of the equation

$$\text{Sharpness of resonance, } S = \frac{\sqrt{I_r^2 - I^2}}{\pm \frac{(f^2 - f_r^2)}{f f_r}}$$

By using some form of this equation many values of S may be computed, depending on what ratio of the radio-frequency current (I) in the circuit at an off-resonance point to the current in the same circuit (I_r) at resonance is chosen. For simplicity, the measurements may be arbitrarily limited to the condition that the square of the off-resonance current (I^2) is one-half the square of the resonance current (I_r^2); thus the numerator is reduced to unity; for this condition the sharpness of resonance is denoted by $S_{\frac{1}{2}}$. When the resonance curves are not symmetrical with respect to the vertical axis passing through the apex of the curve, values of $S_{\frac{1}{2}}$ are computed for both sides of the resonance frequency.

(3) Convenience of operation and simplicity of manipulation, in order that persons not highly trained nor conversant with the details of the circuits used may still be able to operate the receiving set satisfactorily.

(4) Effectiveness in covering the range of frequencies used by the transmitting stations which it is desired to receive.

(5) Substantial construction, in order to remain in serviceable condition in spite of rough handling which may be received during shipment and use.

Besides describing the methods, the paper gives a summary of data from measurements of sensitivity and selectivity or sharpness of resonance of twenty-eight receiving sets of various types, made by fifteen manufacturers using the methods described in this paper.

The Application of Oscillating Valve Circuits to the Precise Measurements of Certain Physical Quantities. J. E. P. WAGSTAFF. (*Phil. Mag.*, Jan., 1924.)—Two oscillating valve circuits, each producing oscillations of frequency approximately equal to a million, were joined to a loud-speaking telephone. This emitted a note when acted on by the waves in the two circuits. "A footfall overhead, the slamming of a door in a distant part of the building, is sufficient to produce a small change (usually one or two beats per second) in the frequency of oscillation. Consequently it has been necessary to do

most of the work described in this paper between the hours of midnight and 3 A.M., when conditions are as nearly perfect as possible." The beats here mentioned were due to the interaction of the note from the loud-speaking telephone and that emitted by a third valve circuit having capacities and inductances so large as to produce an audible note in a telephone in its anode circuit.

The dielectric constant of air was measured by this apparatus. A condenser of small size in which the air pressure could be varied was joined to one of the two interconnected circuits. As the air pressure changed the frequency of the oscillations correspondingly changed, and the number of beats per second made with the third circuit was heard to change. From the relation between the changes of pressure and of frequency the dielectric constant was computed to be from 1.0005976 to 1.0006817 according to the frequency employed.

Magnetic susceptibility can well be measured by this apparatus. A brass rod moves in a vertical plane about a horizontal axis, much nearer to one end of the rod than to the other. From the distant end of the rod a scale-pan for calibration purposes is suspended and from this there hangs a cylinder of bismuth, resting with its axis vertical between the pole pieces of an electromagnet where the magnetic lines of force are horizontal. The other end of the horizontal rod carries the lower plate of a condenser joined to one of the oscillating circuits. The upper and fixed plate of the condenser is a millimetre above the movable one. The valve circuits were adjusted so that slow beats were produced. The time for fifty beats was measured, the current was then sent through the electromagnet and again the time for fifty beats was taken. This was not the same as at first because the bismuth, a diamagnetic substance, was urged upwards. This moved the far end of the brass rod downward, thus increasing the distance between the plates of the condenser and in consequence varying the frequency in the circuit. By the use of the scale-pan the force acting on the bismuth was found to equal .0137 gram. From the dimensions of the rod and the strength of the field the susceptibility of bismuth was calculated to be about -13×10^{-6}. The apparatus was also used in determining the strength of the field.

An interesting application of the apparatus was made in obtaining an answer to the question, "Does the surface tension of mercury change when an electric charge is given to the liquid?" A drop of mercury formed the lower plate and a metal plate one-fifth mm. above the drop the upper plate of a condenser joined to the oscillating circuit. When the liquid was charged to 200 volts not the slightest change in the number of beats could be detected. Had the surface tension changed at all the curvature of the drop would have changed and the capacity would have been slightly different with an effect upon the number of beats. As no change was noted, it is concluded "that no variation in the surface tension of the mercury resulted when the drop was charged to a potential of 200 volts." G. F. S.

NOTES FROM THE U. S. BUREAU OF CHEMISTRY.*

EFFECT OF BROWN AND BLUE PRINT SENSITIZING MATERIALS ON PHYSICAL PROPERTIES OF PAPER.[1]

By T. D. Jarrell and F. P. Veitch.

[ABSTRACT.]

HIGH-QUALITY papers coated with brown print materials, now customarily used in this country, deteriorate with great rapidity at 65 per cent. relative humidity and 70° F. Coated paper kept under these conditions for two weeks has less than one-fourth the folding endurance of the uncoated paper. Therefore, brown print paper should be printed and developed within a week after it is coated.

The wet strength of coated brown print paper, stored at 65 per cent. relative humidity and 70° F., is greater than that of the uncoated for at least sixteen weeks.

Papers of good quality coated with blue print materials, now customarily used in this country, can be kept at least four months at 65 per cent. relative humidity and 70° F. without deterioration.

Long washing after developing (up to fifteen minutes), or machine drying for one-half minute at from 170° F. to 270° F., did not affect the brown and blue print papers examined.

CHANGES IN HYDROGEN-ION CONCENTRATION PRODUCED BY GROWING SEEDLINGS IN ACID SOLUTIONS.[2]

By Jehiel Davidson and Edgar T. Wherry.

[ABSTRACT.]

WHEAT seedlings were grown in solutions of hydrochloric, nitric, sulphuric, phosphoric, formic, acetic, oxalic, succinic, benzoic, and phthalic acids. The changes in reaction produced by their growth, recorded at certain intervals, show that:

Of the inorganic acids, the greatest changes were produced in nitric acid at early stages of growth of the seedlings and in phosphoric acid at later stages. Phosphorus and nitrogen being the most essential elements of plant growth contained in the acids

* Communicated by the Chief of the Bureau.

[1] Published in *Paper Trade J.*, **78** (April 24, 1924) : 45.

[2] Published in *J. Agr. Research*, **27** (Jan. 26, 1924) : 207.

used, it may be concluded that the changes in initial reaction produced by plants in the medium in which they grow, are due to absorption rather than to neutralization.

The previous growth of the experimental seedlings in nutrient solutions deficient in acid-forming elements diminished their ability to decrease the acidity of the acid solutions. Apparently the deficiency of the previous nutrient solutions produced functional disturbances in the seedlings.

The greatest changes from the initial reactions were produced in the solutions of the organic acids. This, however, may have been due partly or wholly to microbiological activity and needs to be studied further.

THE CHEMICAL COMPOSITION OF SESAME OIL.[1]

By George S. Jamieson and Walter F. Baughman.

[ABSTRACT.]

THE oil examined was expressed in the Oil, Fat, and Wax Laboratory by means of an expeller from Chinese yellow sesame seed. A light yellow oil with a slight, pleasant taste, having the following chemical and physical characteristics, was obtained:

Specific gravity 25°/25°	0.9187
Refractive index 20°	1.4731
Acid value	1.4
Iodine number (Hanus)	110.8
Saponification value	189.3
Unsaponifiable matter (per cent.)	1.73
Acetyl value	9.8
Reichert-Meissl number	0.11
Polenske number	0.15
Saturated acids (per cent.)	12.2
Unsaturated acids (per cent.)	81.2

This oil was found to have the following composition:

		Per cent.
Glycerides of	Oleic acid	48.1
	Linolic acid	36.8
	Palmitic acid	7.7
	Stearic acid	4.6
	Arachidic acid	0.4
	Lignoceric acid	0.04
Unsaponifiable matter		1.7

[1] Published in *J. Am. Chem. Soc.*, 46 (March, 1924): 775.

NOTES FROM THE U. S. BUREAU OF MINES.*

METHODS FOR RECOVERY OF PLATINUM, IRIDIUM, PALLADIUM, GOLD, AND SILVER FROM JEWELERS' WASTE.

By C. W. Davis.

BECAUSE of the extensive use of platinum and white gold in jewelry, the Bureau of Mines has received numerous inquiries concerning the recovery of the valuable metals from scrap, filings, sweeps, and other waste. Although persons who have had only a little laboratory experience can use some of the methods, the separation and recovery of the platinum metals is intricate and should be performed only by a skilled chemist. For this reason it is advisable either to employ such a chemist or to ship the material to one of the many refiners of platinum.

Platinum, gold, and silver, and alloys containing two or more of the metals mentioned here—platinum, iridium, gold, palladium, silver, copper, nickel, zinc, and manganese—are likely to be present in material from which the jeweler wishes to recover the valuable metals. Certain mixtures of these metals and alloys may be treated more easily than others; consequently the methods for the recovery of the metals are separated into classes. Where group separations only are required, any or all of the valuable metals may be present either free or alloyed, except that if alloyed the alloy or alloys containing one or more of the metals platinum, iridium, and palladium must be free from gold, silver, or appreciable amounts of copper and zinc; and the alloy or alloys containing gold, silver, and base metals must be free from platinum, iridium, and palladium. The separation will recover platinum, iridium, and palladium in one group, and gold, silver, and base metals in another group. This method depends on the fact that gold, silver, and base metals alloy directly with mercury to form amalgams, but the other metals form an alloy only under certain conditions, such as in the presence of zinc amalgam, acid, and copper sulphate. Zinc or tin must be absent, as an amalgam would be formed with them and the platinum-group metals would go with the amalgam.

* Communicated by the Director.

The less complex methods apply when the valuable metals are not alloyed either with themselves or with the base metals. Further details will be found in Technical Paper No. 342, just published by the bureau.

FRACTIONAL "EDUCTION" OF OIL FROM OIL SHALE.

By Martin J. Gavin and Lewis C. Karrick.

THE theory of fractional "eduction" of oil from oil shale has been used by several inventors as the basis for the design of retorts for the production of oil from the oil shales of the United States. It is based on the belief that when an oil shale is heated it produces light, low-boiling oils at low temperatures, and increasingly heavier, higher-boiling oils as the temperature of retorting rises. The theory evidently had its origin in the fact that crude petroleum may be fractionally distilled, yielding light oils at low distillation temperatures, and heavier oils as the distillation temperature increases. Thus, gasoline is produced from petroleum at relatively low temperatures, kerosene at a higher temperature, and heavier products such as gas-oil and lubricating distillates at still higher temperatures.

The writers have examined several oil-shale retorts designed to take advantage of the belief that crude, and in certain cases refined, gasoline can be recovered from the shale at low temperatures; kerosene at higher temperatures, and so on. Such retorts are usually designed to operate continuously, and have several vapor-pipes leading from them at differing intervals and passing to separate condensers. Designers of such retorts expect that the distillate from the low-temperature parts of the retort will be gasoline, and that as the shale passes to zones of increasingly higher temperatures, the vapor lines leading from these zones will remove increasingly heavier products such as kerosene, gas-oil, etc.

Experimental work on retorts of this nature has in many cases seemed to confirm the theory of "fractional eduction." Light, but by no means fully refined, oils have been recovered from the cooler parts of the retort, and heavier oils from the parts at higher temperatures. The writers during and after an experimental run examined one retort which was apparently fractionating the oil direct from the shale. There were three vapor lines set at different levels in the retort, and each yielded a distillate the specific gravity

of which was roughly in proportion to the temperature of the shale nearest the vapor offtake. It was found, however, that the shale was not fractionally "educing" its oil but that the retort was acting as its own fractionating condenser. The shale actually was producing no oil at all in that part of the retort which was delivering the lighter distillates. The crude oil vapors being produced at a higher temperature, lower down in the retort, rose in the retort and naturally came in contact with cooler retort walls and cooler shale. These cool surfaces condensed the heavier parts of the vapor, leaving to pass out through the vapor line only that portion of the vapor which could exist in vapor state at the lower temperatures. The temperature in that part of the retort from which the lighter products were being condensed was at least 100° C. below that at which the first oil could be produced from the particular shale undergoing treatment. The work is described in more detail in Serial 2588 of the bureau.

TESTS OF A COMMERCIAL SOLUTION USED TO REDUCE THE HAZARD OF CO POISONING IN GARAGES.

By A. C. Fieldner and W. P. Yant.

In its investigations relative to reducing the hazards of carbon monoxide poisoning, the Bureau of Mines recently had its attention drawn to a compound sold under a trade name for which claims are reported to have been made regarding its efficacy in reducing that type of poisoning in automobile garages. Accordingly, some of the product was obtained and analyzed for its power to absorb, oxidize, or destroy carbon monoxide.

It was found that the "compound" does not absorb carbon monoxide from garage atmospheres, consequently the spraying of this solution around garages does not decrease the hazard or danger of poisoning from the inhalation of exhaust gases from automobiles. Details of the tests are shown in Serial 2594, free on application to the Bureau of Mines.

THE PRODUCTION OF LIME FROM SMALL STONE.

By W. M. Myers.

Profitable disposal of limestone, too small for making lime in the shaft kiln, is one of the most urgent problems in the lime industry. Reports from thirty-one quarries show that the pro-

portion of small stone varies from 2 to 40 per cent., averaging 18 per cent. of gross production. Modern methods of heavy blasting and the use of crushers tend to increase the proportion of small stone, also a growing movement toward underground operation, which in general produces more fines than open-pit work.

There are several markets in which small stone may be sold, but in these markets competition is generally keen and prices low. The lime producer's chief product is lime, he understands its manufacture and marketing, and it brings him a greater return than can be obtained from most limestone by-products. For these reasons it is evident that the conversion of small stone into lime is the most desirable means of disposal.

The production of lime from small stone is difficult, and until the rotary kiln was adapted for this purpose little success was achieved. Any method for calcining small stone in the shaft kilns now in use, which could be installed at slight expense, without involving extensive alteration or complete scrapping of the present equipment, would be a great boon to the lime industry. Many experiments have been made attempting to calcine small stone in the shaft kiln, but to date none of them has shown any great promise. An inconsiderable percentage of small stone two to three inches in dimension can be calcined in the shaft kiln when mixed with the larger "one-man" stone. If small stone is introduced into the kiln in appreciable quantity, a marked retardation in draft is noted.

The opinion has been expressed that small stone might be calcined in the shaft kiln, provided it were accurately sized so that the actual combustion chamber or void space would be the same as in the larger stone. An experiment carried out by the Rockland and Rockport Lime Company at Rockland, Maine, demonstrated that the possibilities of burning closely sized small stone in the shaft kiln are slight unless the design and operation of the kiln be radically changed.

Some attempts to use forced and induced draft have not been successful. The employment of forced draft produced by the introduction of a blast of air at the fire boxes has resulted in a concentration of intense heat at this point which overburned the lime directly before the blaze without penetrating to the uncalcined stone in the higher levels of the kiln.

The calcination of small stone is further complicated by the fact that the lime may crumble readily, producing considerable fine material in the kiln. The tendency of some limes to crumble is so excessive as to occasionally make trouble even when only large stone is used. In addition to this the stone itself may shatter when it is undergoing its first gradual heating in the kiln, particularly if any incipient fractures are present. These factors increase the retardation of the draft and add to the difficulties of calcining small stone.

The possibilities of altering the shaft kiln so that the small stone may be calcined seem remote at best.

It is probable that the rotary kiln will be much more extensively used in the future than now, and will be used more and more as a primary producer of lime, calcining the whole quarry output rather than confining it to the small stone unsuitable for shaft kiln operation. Further details are presented in Serial 2596.

The Jubilee of the Physical Society of London. (*Nature*, March 29, 1924.)—The fiftieth anniversary of the foundation of this society was celebrated on March 20–21, 1924. Delegates were present from Great Britain and from other countries. The Duke de Broglie delivered the Guthrie lecture on the effects of high-frequency radiation and Sir Richard Paget in the evening discussed the nature of speech. At his behest the "cheirophone" enlarged its repertoire by enunciating the appropriate names, "Oliver Lodge" and "Vernon Boys."

"The Society grew originally out of the courses for science teachers held at South Kensington, and Guthrie chose for himself the minor post of demonstrator. Prof. J. A. Fleming, who had read the first paper to the Society fifty years earlier almost to an hour, said that the Society was founded in the belief that incomplete researches might give and receive stimulus if submitted to discussion. Some discountenance was met with, from Clerk Maxwell amongst others, and the attitude of the Royal Society was at first unsympathetic, but the above aim has been vindicated by the publication of progressive series of papers in the *Proceedings*. Guthrie himself showed in 1874 that red-hot iron loses negative more quickly than a positive charge, and was therefore the first to notice thermionic phenomena."

"Sir Arthur Schuster gave some amusing reminiscences of Joule (whose paper on the C^2R law the Royal Society refused to publish), of Balfour Stewart, Stokes and Boltzman. Prof. H. E. Armstrong regretted the passing of the popular science of Tyndall, and mentioned that Guthrie was the inventor of what is now known as mustard gas. Sir Oliver Lodge claimed that the relativity postulate of an absolute velocity implies an ether, and hinted at a psychic function for the

latter. He quoted some 'queries' of Newton, one of which foreshadowed the modern identification of mass with energy."

The accompanying exhibition was of high interest. The way in which Boys got his first fused quartz products was shown and fibres of this substance were made by the shooting process. "Other series showed the development of the oscillograph, of the induction motor (the earliest demonstration of a rotating field having been given to the Society in 1879), of vacuum apparatus (the original McLeod gauge being shown), of resistance thermometry, the photo-electric cell, the moving coil, galvanometer, etc." "Sir Joseph Thomson, proposing the 'Visitors' in a witty speech, said that, although not a student of physics while at Cambridge, H.R.H. the Duke of York had excelled in the technique of that branch of applied physics which relates to the effect of rotation on the track of a sphere moving through an elastic fluid." G. F. S.

The Resistance of Air to Falling Spheres. R. G. LUNNON. (*Phil. Mag.*, Jan., 1924.)—"Measurements have been made of the times of fall of spheres of various sizes and densities for distances up to 600 metres in the shafts of four coal mines in this district (Newcastle-on-Tyne), and also, for shorter distances, in the tower and laboratories of this college (Armstrong)." Polished steel balls were mainly used. These had fifteen different diameters, the range being from .24 to 1.83 cm. The relations for falling bodies that hold only *in vacuo* are so ingrained in our thought that the data here presented will give a wholesome corrective by calling attention to the modifications introduced by the presence of the retarding air. For example, a sphere of diameter .476 cm. in 1, 2, 4, 6, 8, 10, and 12 seconds from the start falls through distances, respectively, equal to 4.3, 16.2, 59, 110, 166, 228 and 297 m. There is a great departure from the vacuum law that the distances are proportional to the squares of the time. The terminal velocity is given as 33.1 m. per second. The larger the sphere is the greater is its terminal velocity and the more closely does its velocity at any time agree with that of a body falling in a vacuum.

On the assumption that the force retarding the fall of a sphere is the sum of two forces, one of which is in proportion to the square of the velocity while the other varies as the actual acceleration, the author deduces a formula which fits his measured results quite well, especially after the ball has been falling for more than two seconds. From this formula may be deduced the conclusion drawn a decade ago by Shakespear "that if different balls fall a certain distance in the same time, then their weights are in proportion to their areas." The formula likewise answers affirmatively this question, "If two spheres of different diameters are weighted so as to fall a certain distance in equal times, will they take equal times for all other distances?" G. F. S.

THE FRANKLIN INSTITUTE.

(*Proceedings of the Stated Meeting held Wednesday, May 21, 1924.*)

[ABSTRACT.]

HALL OF THE FRANKLIN INSTITUTE,

PHILADELPHIA, May 21, 1924.

MR. WM. C. L. EGLIN, President, *in the Chair.*

On motion, duly seconded, the minutes of the previous meeting were approved as published in the JOURNAL.

The Chairman announced that the annual presentation of The Franklin Medal, the Institute's highest award for recognition of distinguished scientific and technical achievements, would be made, and recognized Dr. Joseph S. Ames, Dean, the Johns Hopkins University, Baltimore, Maryland, who made a statement relative to the work of Sir Ernest Rutherford, F.R.S., Cavendish Professor of Physics, Cambridge University, Cambridge, England, recently awarded The Franklin Medal in recognition "of long-continued and fruitful researches, signally contributing to the present state of knowledge of the elements, their constitution and relationships."

H. G. Chilton, Esquire, C.M.G., Councellor of the British Embassy, Washington, D. C., representing His Britannic Majesty's Government, was then presented to the Chairman and received from him The Franklin Medal, accompanying Certificate and Certificate of Honorary Membership in the Institute, awarded to Sir Ernest Rutherford.

Mr. Chilton expressed for his Government its appreciation of the high honour conferred upon his distinguished countryman.

Dr. Frank J. Sprague, of New York City, was then recognized and described the work of Dr. Edward Weston, of Newark, New Jersey, who had been awarded The Franklin Medal in recognition "of discoveries and inventions in the field of electricity immeasurably fruitful and fundamentally contributory to the establishment of the electric art."

Due to the illness of Doctor Weston, Dr. L. H. Baekeland, President of the American Chemical Society, was presented to the Chairman and received from him The Franklin Medal, accompanying Certificate and Certificate of Honorary Membership in the Institute for Doctor Weston.

The paper, "Early Days in Radio-activity," prepared for presentation at this meeting by Sir Ernest Rutherford, was read by Dr. Joseph S. Ames.

Adjourned.

R. B. OWENS, *Secretary.*

(A full account of the meeting will appear in a later issue of the JOURNAL.)

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, May 7, 1924.)

HALL OF THE INSTITUTE,
PHILADELPHIA, May 7, 1924.

The following report was presented for final action:

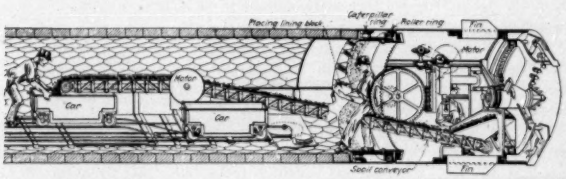
No. 2823: Sheen Tunneller. The Edward Longstreth Medal to Milton Roy Sheen, Esquire, of Philadelphia, Pennsylvania.

The Committee having in charge the investigation of this tunneller reported in part as follows:

The Sheen Expansion Method of Tunnel Construction is accomplished by a machine designed for the construction of masonry-lined tunnels, mains, and conduits in one operation.

In this machine a combination of a boring head, a circular protecting shield and a rotating tail-cam for expanding and forcing the lining blocks of the tunnel into place and continuously advancing the cutting head and shield, marks its difference from other types of tunnelling machines.

A sectional view of the Sheen machine is shown on the following cut.



Its operation may be described briefly as follows:

A shaft is excavated to the depth it is desired to construct the tunnel and the machine is lowered into the excavation. The earth excavated by the motor-driven cutting head is forced back through a funnel-shaped opening onto a conveyor belt from where it is taken back in small cars to the shaft and hoisted out of the way. The blocks used in lining the tunnel are carried forward to the work in the same cars that are used for the excavated earth. These blocks are placed in the tail of the machine and forced into place by the rotating tail-cam which also serves to force the cutting-head forward.

R. B. OWENS,
Secretary.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

(Stated Meeting, Board of Managers, May 14, 1924.)

RESIDENT MEMBERS.

GUILLIAEM AERTSEN, Esquire, Engineer, Bethlehem Steel Company, 1637 Widener Building, Philadelphia, Pennsylvania.

G. W. AKERLOW, M.E., Philadelphia Electric Company, Philadelphia, Pennsylvania.

GEORGE P. MCARTHUR, Esquire, Mechanical Engineer, Joseph Oat and Sons, 232 Quarry Street, Philadelphia, Pennsylvania.

WALTER C. WAGNER, E.E., Engineer, Philadelphia Electric Company, 1000 Chestnut Street, Philadelphia, Pennsylvania.

NON-RESIDENT MEMBERS.

BENJAMIN WILEY SANDS, Research Worker, Brunswick, Maryland.

LEWIS E. SAUNDERS, B.S., Manager, Research Laboratories, Norton Company, Worcester, Massachusetts.

ASSOCIATE MEMBERS.

KARL F. OERLEIN, Esquire, Student, 1246 North Twelfth Street, Philadelphia, Pennsylvania.

CHANGES OF ADDRESS.

MR. JOHN MARKLE, Attention of Mr. A. B. Jessup, Jeddo, Pennsylvania.

ALLEN I. MYERS, Esquire, 316 North Chester Road, Swarthmore, Pennsylvania.

MR. C. B. NEBLETTE, 444 Mountain Avenue, S.W., Roanoke, Virginia.

MR. T. MORRIS PEROT, JR., 605 Chestnut Street, Philadelphia, Pennsylvania.

CHARLES H. UMSTEAD, Esquire, 331 C Street, N. W., Washington, District of Columbia.

MR. CHARLES A. WILLIAMS, 119 North College Avenue, Oxford, Ohio.

GEORGE M. YORKE, Esquire, 195 Broadway, New York City, New York.

NECROLOGY.

Mr. William Griffith, Coal Exchange, Scranton, Pennsylvania.

Mr. T. Commerford Martin, 32 Grove Avenue, Larchmont Manor, New York.

Dr. Thomas Corwin Mendenhall, Ravenna, Ohio.

LIBRARY NOTES.

PURCHASES.

American Annual of Photography, 1924. 1923.

BEARN, J. GOULD.—Chemistry of Paints, Pigments and Varnishes. 1924.

BLUM, WM., and HOGABOOM, G. B.—Principles of Electroplating and Electroforming. 1924.

GIBBS, WM. E.—Clouds and Smokes. 1924.

GOODRICH, W. FRANCIS.—Pulverized Fuel. 1924.

GOULD, RUPERT T.—Marine Chronometer. 1923.

HARBORD and HALL.—Metallurgy of Steel. Vols. 1 and 2. 1923.

KAYSER and KONEN.—Handbuch der Spectroscopie. Vol. 7, part 1. No date.

LANGE, OTTO.—Chemisch-Technische Vorschriften. Vol. 3. 1923.

RÜDISÜLE, A.—Nachweis, Bestimmung und Trennung der Chemischen Elemente. Vol. 6, parts 1 and 2. 1923.

SEARLE, ALFRED B.—Chemistry and Physics of Clays and Other Ceramic Materials. 1924.

GIFTS.

American Engineering Company, Lo-Hed, Mono-Rail Electric Hoist. Philadelphia, Pennsylvania, 1923. (From the Company.)

American Institute of Electrical Engineers, Yearbook, 1924. New York City, New York, 1924. (From the Institute.)

American Institute of Mining and Metallurgical Engineers, Incorporated, Production of Petroleum in 1923. New York City, New York, 1924. (From the Institute.)

American Institute of Mining and Metallurgical Engineers, Transactions, Vol. 69. New York City, New York, 1923. (From Mr. Wm. R. Webster.)

American Metal Hose Company, Flexible Metal Hose. Waterbury, Connecticut, 1920. (From the Company.)

American Society of Civil Engineers, Transactions, Vol. 86. New York City, New York, 1923. (From Mr. Wm. R. Webster.)

American Society of Mechanical Engineers, Transactions, Vol. 44, 1922. New York City, New York, 1923. (From Mr. Wm. R. Webster.)

American Society of Mechanical Engineers, Yearbook, 1924. New York City, New York, 1924. (From the Society.)

Amsler, Alfred J., and Company, Catalogues, Torsion Dynamometers, Testing Machines. Schaffhouse, Switzerland, no date. (From the Company.)

Anchor Concrete Machinery Company, Handbook on Concrete Products Machinery. Columbus, Ohio, no date. (From the Company.)

Austin Western Road Machinery Company, Catalogue No. 24. Philadelphia, Pennsylvania, 1924. (From the Company.)

Babcock and Wilcox Company, Steam, Its Generation and Use. New York City, New York, 1923. (From the Company.)

Baird Machine Company, Brief Bits 'Bout Baird Machinery; Bulletin 201. Bridgeport, Connecticut, 1923 and 1924. (From the Company.)

Baker University, Catalogue, 1924–1925. Baldwin City, Kansas, 1924. (From the University.)

Baldwin Chain and Manufacturing Company, Catalogue G, 1924. Worcester, Massachusetts, 1924. (From the Company.)

Baldwin-Wallace College, Catalogue, 1923–1924. Berea, Ohio, 1924. (From the College.)

Belden Manufacturing Company, Catalogue No. 9, Electrical Wire Cable and Cordage. Chicago, Illinois, no date. (From the Company.)

Bethlehem Steel Company, Catalogue TS, Twin-Span Turntables. Bethlehem, Pennsylvania, 1924. (From the Company.)

Beyer, Peacock and Company, Limited, Locomotives for all Gauges and Duties. Manchester, England, no date. (From the Company.)

Bishop, J., and Company, Technical and Industrial Platinum. Malvern, Pennsylvania, 1923. (From the Company.)

Blaw-Knox Company, Catalogue No. 24, Clamshell Buckets. Pittsburgh, Pennsylvania, 1923. (From the Company.)

Bond, Charles, Company, Catalogue No. 44. Philadelphia, Pennsylvania, 1924. (From the Company.)

Borden Company, Beaver Pipe Cutting and Threading Tools, Catalogue No. 17. Warren, Ohio, 1924. (From the Company.)

Boston and Maine Railroad, Ninety-first Annual Report for the Year Ended December 31, 1923. Boston, Massachusetts, 1924. (From the Railroad.)

Boye and Emmes Machine Tool Company, Engine Lathes. Cincinnati, Ohio, no date. (From the Company.)

Broadbent, Thomas, and Sons, Limited, Catalogue, The "Broadbent" Patent Centrifugal Clutch. Huddersfield, England, no date. (From the Company.)

Brownell Company, Bulletin No. B 1. Dayton, Ohio, no date. (From the Company.)

Bucyrus Company, Bulletins B-201, F-302, C-304. South Milwaukee, Wisconsin. (From the Company.)

Byers Machine Company, "Bear Cat" Crane Model 1. Ravenna, Ohio, no date. (From the Company.)

Cambridge and Paul Instrument Company of America, Incorporated, Bulletins, List Nos. 157, 158, 194A, 195A, 906. New York City, New York, no date. (From the Company.)

Canada, Dominion Water Power Branch, Annual Report, 1922–1923. Ottawa, Canada, 1924. (From the Branch.)

Capital University, Annual Catalogue, 1923–1924. Columbus, Ohio, 1924. (From the University.)

Carleton College, Catalogue, 1923–1924. Northfield, Minnesota, 1924. (From the College.)

Carron Company, Catalogues, Haulage Gear, Ships' Side and Saloon Lights, Ships' Fittings, Steering Gear, Ships' Cooking Apparatus, Drop Forgings. Falkirk, England, 1920–1923. (From the Company.)

Central College and Howard Payne Junior College, Catalogue, 1924. Fayette, Missouri, 1924. (From the College.)

Century Electric Company, Bulletin No. 36, Alternating and Direct Current Fans. St. Louis, Missouri, no date. (From the Company.)

Champion Pneumatic Machinery Company, Catalogue No. 16, Air Compressors. Chicago, Illinois, no date. (From the Company.)

College of the City of New York, Annual Register, 1923–1924. New York City, New York, 1924. (From the College.)

Columbia University, Announcement, 1924–1925, Courses in Optics and Optometry; Summer Session, 1924. New York City, New York, 1924. (From the University.)

Connecticut College, Catalogue, 1923–1924. New London, Connecticut, 1924. (From the College.)

Connecticut, State Department of Health, Report, 1921–1922. Hartford, Connecticut, 1922. (From the Department.)

Construction Machinery Company, Catalogue No. 35, Wonder Concrete Mixers. Waterloo, Iowa, no date. (From the Company.)

Copperweld Steel Company, Engineering Data Bulletin. Braddock, Pennsylvania, no date. (From the Company.)

Cramp, Wm., and Sons Ship and Engine Building Company, The Larner-Johnson Valve and Engineering Company, Bulletin No. 3—Hydraulic Valves. Philadelphia, Pennsylvania, 1923. (From the Company.)

Dakota Wesleyan University, General Catalogue, 1924–1925. Mitchell, South Dakota, 1924. (From the University.)

Dana, R. W., The Institution of Naval Architects and Its Work. London, England, 1924. (From the Author.)

Davidson, M. T., Company, Bulletins 101, 102, and 103. New York City, New York, no date. (From the Company.)

Denver Fire Clay Company, Catalogue No. 10, Scientific Apparatus, Refractories. Chicago, Illinois, 1923. (From the Company.)

Digestive Ferments Company, Endocrines, Enzymes and Protein Compounds. Detroit, Michigan, no date. (From the Company.)

Dimmick, R. B., Metallurgy of Modern Commercial Iron Sheets. Canton, Ohio, no date. (From the United Alloy Steel Corporation.)

Dobbins Core Drill Company, Incorporated, Catalogue No. 16. Long Island City, New York, no date. (From the Company.)

Dorr Company, Bulletins. New York City, New York, 1922–1924. (From the Company.)

Duff Manufacturing Company, Catalogue No. 106. Pittsburgh, Pennsylvania, 1923. (From the Company.)

Dust Recovering and Conveying Company, Bulletin No. 10. Cleveland, Ohio, 1924. (From the Company.)

Electric Storage Battery Company, Bulletin No. 196. Philadelphia, Pennsylvania, 1924. (From the Company.)

Elliott Company, Lagonda Tube Cleaners. Philadelphia, Pennsylvania, 1923. (From the Company.)

Etna Machine Company, Swaging Machines. Toledo, Ohio, no date. (From the Company.)

Fitchburg Steam Engine Company, Catalogue. Fitchburg, Massachusetts, no date. (From the Company.)

Food Research Institute, Announcement, 1923–1924. Stanford University, California, 1924. (From the Institute.)

Fowler, John, and Company, Limited, General Catalogue, Part IV, Section A, No. 1, Section B, No. 1. London, England, no date. (From the Company.)

Foxboro Company, Bulletin No. 96—1. Foxboro, Massachusetts, no date. (From the Company.)

Franklin College, Catalogue, 1924–1925. Franklin, Indiana, 1924. (From the College.)

Gedge-Gray Company, Catalogue No. 14, Feeders, Sifters and Mixers. Lockland, Ohio, no date. (From the Company.)

General Electric Company, Thirty-second Annual Report, 1923. Schenectady, New York, 1923. (From the Company.)

Gillis and Geoghegan, Incorporated, the G and G Telescopic Hoist. New York City, New York, 1923. (From the Company.)

Girtanner Engineering Corporation, Standardized Steam Ash Conveyor. New York City, New York, no date. (From the Corporation.)

Gold Car Heating and Lighting Company, Catalogue No. 24. Brooklyn, New York, 1924. (From the Company.)

Griscom-Russell Company, Bulletin No. 1216; Raw Water Distilling Plants for Producing Distilled Boiler Feed Make-up Water, by Joseph Price. New York City, New York, no date. (From the Company.)

Hartford Steam Boiler Inspection and Insurance Company, The Locomotive, Vol. 34. Hartford, Connecticut, 1922–1923. (From the Company.)

Henry and Wright Manufacturing Company, Viktor Drilling Machines. Hartford, Connecticut, no date. (From the Company.)

Hill Clutch Machine and Foundry Company, Catalogue No. 11, Power Transmission Machinery. Cleveland, Ohio, no date. (From the Company.)

Holbeck Engineering Company, Pulverized Coal Applied to the Malleable Iron Industry. Cleveland, Ohio, no date. (From the Company.)

Hood College, Catalogue, 1923–1924. Frederick, Maryland, no date. (From the College.)

Hunter Saw and Machine Company, Catalogue E. Pittsburgh, Pennsylvania, no date. (From the Company.)

Independent Pneumatic Tool Company, Thor Catalogue No. 11. Philadelphia, Pennsylvania, no date. (From the Company.)

India, Meteorological Office, Monthly Rainfall for 1922. Calcutta, India, 1923. (From the Office.)

Ingersoll-Rand Company, Form No. 3050, Compressors. New York City, New York, 1924. (From the Company.)

Insley Manufacturing Company, Catalogue No. 46. Indianapolis, Indiana, no date. (From the Company.)

Instituto y Observatorio de Marina, Almanaque Nautico, 1925. San Fernando, Spain, 1923. (From the Institute.)

Interstate Equipment Corporation, Automatic Aerial Tramways. New York City, New York, no date. (From the Corporation.)

Iron and Steel Institute, Journal, Vol. 108, No. 11; Charter, By-Laws and List of Members and Associates. London, England, 1923 and 1924. (From the Institute.)

Johns-Manville, Incorporated, Service to Railroads. New York City, New York, 1923. (From the Company.)

Jointless Fire Brick Company, Refractories and Furnace Design. Chicago, Illinois, 1923. (From the Company.)

Jones, W. A., Foundry and Machine Company, Catalogue No. 27. Chicago, Illinois, 1923. (From the Company.)

Kansas State Board of Agriculture, Report for the Quarter Ending December, 1923. Topeka, Kansas, no date. (From the Board.)

Lake Erie College, Catalogue, 1923–1924. Painesville, Ohio, 1924. (From the College.)

Lawrence College, Catalogue, 1923–1924. Appleton, Wisconsin, 1924. (From the College.)

Leeds and Northrup Company, Catalogue Nos. 75 and 87. Philadelphia, Pennsylvania, 1921 and 1924. (From the Company.)

Lincoln Memorial University, Exponent, Vol. 5, No. 1, Summer Bulletin. Harrogate, Tennessee, 1924. (From the University.)

Linfield College, Annual Catalogue, 1923–1924. McMinnville, Oregon, 1924. (From the College.)

Lombard Governor Company, Bulletin 21-S, Diesel Oil Engines. Ashland, Massachusetts, no date. (From the Company.)

Louden Machinery Company, Catalogue No. 6. Fairfield, Iowa, no date. (From the Company.)

Lucas Furnaces, Antiscale Furnaces. Birmingham, England, no date. (From the Lucas Furnaces.)

Ludlow-Saylor Wire Company, Catalogue No. 46. St. Louis, Missouri, no date. (From the Company.)

Lufkin Rule Company, General Catalogue No. 11. Saginaw, Michigan, no date. (From the Company.)

Lupton's, David, Sons Company, Catalogue No. 11, Service Products. Philadelphia, Pennsylvania, 1922. (From the Company.)

Mancha Storage Battery Locomotive Company, Catalogue, Storage Battery Locomotives. St. Louis, Missouri, no date. (From the Company.)

Marion Machine Foundry and Supply Company, Catalogue No. 50, Soot Blowers. Marion, Indiana, 1923. (From the Company.)

McClave-Brooks Company, Catalogue of Grates, Stokers and Steam Blowers. Scranton, Pennsylvania, 1923. (From the Company.)

Metals Coating Company of America, Metalayer—Schoop Process. Philadelphia, Pennsylvania, no date. (From the Company.)

Milburn, Alexander, Company, Bulletins, Welding and Cutting Apparatus. Baltimore, Maryland, no date. (From the Company.)

Milton College, Catalogue, 1923–1924. Milton, Wisconsin, no date. (From the College.)

Milwaukee Corrugating Company, Catalogue No. 20, Metallic Construction for the Modern Home. Milwaukee, Wisconsin, no date. (From the Company.)

Mississippi State Geological Survey, Bulletin No. 19, The Bauxite Deposits of Mississippi, by Paul Franklin Morse. Jackson, Mississippi, 1923. (From the Survey.)

Missouri Bureau of Geology and Mines, Vol. xvii, Second Series, The Devonian of Missouri, 1922. Rolla, Missouri, 1923. (From the Bureau.)

Mount Union College, Catalogue, 1923–1924. Alliance, Ohio, 1924. (From the College.)

Muhlenberg College, Catalogue, 1924. Allentown, Pennsylvania, 1924. (From the College.)

Municipal College of Technology, The Journal, Vol. 11. Manchester, England, 1923. (From the College.)

National Metal Molding Company, Conduits. Pittsburgh, Pennsylvania, no date. (From the Company.)

National Paving Brick Manufacturers Association, The Construction of Vitrified Brick Pavements. Cleveland, Ohio, 1924. (From the Association.)

National Tube Company, Bulletin No. 1. Pittsburgh, Pennsylvania, no date. (From the Company.)

Nelson, N. P., Iron Works, Incorporated, Fordson Loader. Brooklyn, New York, no date. (From the Company.)

New Orleans, Board of Commissioners of the Port, Twenty-seventh Report as of August 31, 1923. New Orleans, Louisiana, no date. (From the Board.)

New York Belting and Packing Company, Modern Engineering for Power Plant Operators, by W. H. Wakeman. New York City, New York, 1923. (From the Company.)

New York, Ontario and Western Railway Company, Annual Report for Year Ended December 31, 1923. New York City, New York, 1924. (From the Company.)

New York University, College of Arts and Pure Science, Announcements, 1924–1925. New York City, New York, 1924. (From the University.)

New Zealand Geological Survey, Paleontological Bulletin No. 10. Wellington, New Zealand, 1924. (From the Survey.)

New Zealand Institute, Transactions and Proceedings, Vol. 54. Wellington, New Zealand, 1923. (From the Institute.)

Northampton Water Commissioners, Fifty-third Annual Report for the Year Ending November 30, 1923. Northampton, Massachusetts, no date. (From the Commissioners.)

Northern Pacific Railway Company, The Facts about the Northern Pacific Land Grant. St. Paul, Minnesota, 1924. (From the Company.)

Ontario Department of Agriculture, Annual Report, 1922. Toronto, Canada, 1923. (From the Department.)

Pacific Lumber Company, California Redwood Construction Digest. Chicago, Illinois, 1923. (From the Company.)

Packard Electric Company, Booklets on Transformers. Warren, Ohio, no date. (From the Company.)

Page Boiler Company, Catalogue, Water Tube Boilers. Chicago, Illinois, 1920. (From the Company.)

Palo Company, Catalogue C—Apparatus for Industrial and Laboratory Use. New York City, New York, no date. (From the Company.)

Pennsylvania Bureau of Statistics and Information, Report on Productive Industries, Railways, Taxes and Assessments, Waterways and Miscellaneous Statistics for the Year 1921. Harrisburg, Pennsylvania, 1923. (From the Bureau.)

Pennsylvania Geological Survey, Oil and Gas, Vol. 1, Introduction, by Ashley and Robinson; Oil Resources in Coal and Carbonaceous Shales of Pennsylvania. Harrisburg, Pennsylvania, 1922 and 1923. (From the Survey.)

Pennsylvania Railroad System, Record of Transportation Lines for the Year Ending December 31, 1923. Philadelphia, Pennsylvania, 1924. (From the System.)

Proctor and Schwartz, Incorporated, Drying Machinery for Electrical and Special Porcelain. Philadelphia, Pennsylvania, no date. (From the Company.)

Ruggles-Klingemann Manufacturing Company, Catalogue A; Draft Regulation. Salem, Massachusetts, 1922. (From the Company.)

Ryerson, Joseph T., and Son, Incorporated, Bulletins 1301 and 13,001. Chicago, Illinois, no date. (From the Company.)

Safety Car Heating and Lighting Company, Postal Car Lighting, 1924. New Haven, Connecticut, 1924. (From the Company.)

St. Olaf College, Annual Catalogue, 1923–1924. Northfield, Minnesota, 1924. (From the College.)

Sanford Riley Stoker Company, Publications Nos. 45 and 82, Underfeed Stokers; Publication No. 85, Burning Waste Fuels. Worcester, Massachusetts, no date. (From the Company.)

Saxild and Partners, Reinforced Concrete. London, England, no date. (From the Company.)

Schieren, Chas. A., Company, Quality Facts about Belting. New York City, New York, no date. (From the Company.)

Schwartz Sectional System, How to Store and Catalogue Your Laboratory Stock of Chemicals, Reagents, Samples and Specimens. Indianapolis, Indiana, no date. (From the System.)

Sement-Solvay and Piette Coke Oven Company, Limited, Dry Quenching of Hot Coke—Sulzer Process. Sheffield, England, no date. (From the Company.)

Sharples Specialty Company, Super Centrifuge and Continuous Flow Process. Philadelphia, Pennsylvania, no date. (From the Company.)

Smith and Serrell, Bulletin No. 37. Newark, New Jersey, 1923. (From the Company.)

South Australia Department of Mines, Annual Report of the Director of Mines and Government Geologist for 1922. Adelaide, South Australia, 1923. (From the Department.)

South Dakota, Board of Railroad Commissioners of the State, Thirty-fourth Annual Report, for the Fiscal Year Ended June 30, 1923. Pierre, South Dakota, no date. (From the Board.)

South Dakota Geological and Natural History Survey, Bulletin 12 and Circular 14. Vermillion, South Dakota, 1923. (From the Survey.)

Standard Calorimeter Company, Catalogue D, Booklets 101, 103, and 104. East Moline, Illinois, no date. (From the Company.)

Sullivan Machinery Company, Bulletins Nos. 77-B, 70-X, 77-H, 77-K, 78-B. Chicago, Illinois, 1923 and 1924. (From the Company.)

Swenson Evaporator Company, Bulletin E-122, Swenson Evaporators. Chicago, Illinois, no date. (From the Company.)

Tagliabue, C. J., Manufacturing Company, Catalogue No. 905A. Brooklyn, New York, no date. (From the Company.)

Terry Steam Turbine Company, The Terry Turbine, Condensing and Non-condensing. Hartford, Connecticut, no date. (From the Company.)

Thermo Electric Instrument Company, Freas Apparatus. Newark, New Jersey, no date. (From the Company.)

Timken Roller Bearing Company, Seven Booklets on Bearings. Canton, Ohio, 1917–1922. (From the Company.)

Traveling Engineers' Association, Thirty-first Annual Report, 1923. Cleveland, Ohio, no date. (From the Association.)

Tyler, W. S., Company, Catalogue 45, Electric Screens. Cleveland, Ohio, 1922. (From the Company.)

United Fruit Company, The Story of Cuban Sugar. Boston, Massachusetts, 1924. (From the Company.)

United States Bureau of the Census, Fourteenth Census of the United States Taken in the Year 1920, Vol. IV, Population, Occupations. Washington, District of Columbia, 1923. (From the Bureau.)

United States Department of Commerce, Trade Information Bulletin, No. 209, Industrial Machinery Division, Ice-making and Cold-storage Plants in

South America. Washington, District of Columbia, 1924. (From the Department.)

United States Naval Observatory, The American Ephemeris and Nautical Almanac for the Year 1926. Washington, District of Columbia, 1924. (From the Observatory.)

United States War Department, Index-Catalogue of the Library of the Surgeon General's Office, Third Series, Vol. 4. Washington, District of Columbia, 1923. (From the Surgeon General's Office.)

University of Nebraska, College of Engineering, Announcements, 1924–1925. Lincoln, Nebraska, 1924. (From the University.)

University of Vermont, Catalogue, 1923–1924. Burlington, Vermont, 1924. (From the University.)

University of Virginia, Catalogue, 1923–1924. Charlottesville, Virginia, 1924. (From the University.)

Virginia Geological Survey, Bulletin No. xxiv, The Geology and Mineral Resources of Wise County and the Coal-bearing Portion of Scott County, Virginia. Charlottesville, Virginia, 1923. (From the Survey.)

Vulcan Steam Forging Company, Forgings. Buffalo, New York, no date. (From the Company.)

Wabash College, Catalogue, 1924. Crawfordsville, Indiana, 1924. (From the College.)

Western Electric Company, Electrical Supply Yearbook, 1924. Philadelphia, Pennsylvania, 1924. (From the Company.)

Westinghouse Electric and Manufacturing Company, Supervisory Control. East Pittsburgh, Pennsylvania, no date. (From the Company.)

Wickes Boiler Company, Bulletin No. 6, The Utilization of Waste Heat for Steam Generation. Saginaw, Michigan, no date. (From the Company.)

Wyckoff, A., and Son Company, Catalogue No. 44, Wyckoff Wood Pipe, no date. (From the Company.)

Yale and Towne Manufacturing Company, Catalogue No. 21-M. Stamford, Connecticut, no date. (From the Company.)

BOOK REVIEWS.

CHEMICAL SYNONYMS AND TRADE NAMES. A Dictionary and Commercial Handbook. By William Gardner. 271 pages, 8vo. London, Crosby, Lockwood and Son, 1924.

The earliest nomenclature in chemistry was purely arbitrary, names being often given for fanciful reasons, but occasionally bearing a definite relation to source, property, or even composition. Many of these are still in familiar use. Blue vitriol, Epsom salt, cream of tartar and quicklime have common vogue. Others have been lost entirely except to the delver in the ancient lore of the sciences of chemistry and pharmacy. Few of those in these professions to-day would recognize *sal enixum*, Cadet's fuming liquor, vitriolated tartar, the Jovian preparations. Lavoisier was the principal initiator of our modern systematic nomenclature, but the enormous extension of chemistry, especially in the organic field, has outrun his general plan, and to-day, in consequence of the lack of international agreement and even of effective agreement among the

chemists of a given country, systematic nomenclature is but little developed. The pharmacists have been among the most conservative of the groups concerned.

The main object of the systematic name of a chemical compound is to indicate its molecular constitution. In Lavoisier's day, the number of known compounds was comparatively small, especially in the organic field. In the case of organic bodies, considerable reliance was placed on the properties, the classification and description being mainly determined by these. As Wurtz remarked in one of his essays, the older courses on descriptive organic chemistry included in one group the "organic acids," merely because they had the common property of turning blue litmus red, and another group was termed the "alkaloids" because the members thereof had the common property of turning the red litmus back to blue. As synthetic organic chemistry developed, compounds of extreme complexity were introduced with great rapidity and the systematic nomenclature either broke down or produced such sesquipedalian words that their use was impracticable. Under this strain grew up the familiar system of "trade names" often quite as fanciful as any term invented in the middle ages or by the Greek alchemists. Sodium dioxybenzene-azobenzene-parasulphonate is more manageable as "tropæolin O."

The book in hand contains about 14,000 definitions and cross-references. It is a welcome contribution to a department of chemistry that is continually growing and is already vast and complex. The preparation of such a work is really hard labor. The time and trouble expended on it are not easily appreciated by those who have not done such work. The text covers a wide field, not being limited, as might be thought, to organic chemistry, but including names in all departments, even minerals and pharmaceutical preparations. It is, indeed, valuable as a laboratory companion, for information is given concerning many tests and reagents and much of it is interesting reading. It is excellently printed, in clear type, with very distinct lettering, and its production does great credit to the author and publisher. HENRY LEFFMANN.

CHEMISCH-TECHNISCHE VORSHRIFTEN. By Otto Lange. Third, revised and enlarged edition. Fourth volume, including fertilizers, explosives, fodders and foods. xxi–750 pages, 8vo. Leipzig, Otto Spamer, 1924. Price, 45 gold marks.

The present volume finishes the third edition of this great work. Full information as to its general merits and methods has been set forth in previous reviews; it remains merely to say that the fourth volume maintains the excellent character of its predecessors, and the whole work constitutes a most valuable contribution to applied chemistry. A striking and important feature of the last volume is the inclusion of the text on explosives in the section on fertilizers. Many pages are devoted to this phase of chemical industry, and the tie between the two subjects is, of course, the modern development of nitrogen-fixation, in which Germany has made such remarkable strides, duplicating in fact the development of the coal-tar color industry. Chemists interested in the great problem of nitrogen-fixation will turn to the chapter on the Haber process with much curiosity to learn, if possible, whether this book, one of the latest to discuss the topic, will throw a bright light on the problems which have been so difficult to all but Germans. Lately, it has been authoritatively stated that French ammunition makers have taken over the Haber process and practically

acknowledged that the experts of their own and other nations have not succeeded in getting a practical method of operating the synthesis of nitrogen and hydrogen, which constitutes a key industry in the production of explosives and fertilizers. Independence of the Chile nitrates is now a necessity to all countries, not only because these may fail, but because of the long sea haul, which may be easily obstructed by a hostile fleet. Meanwhile the United States shilli-shallies with the Muscle Shoals plant, the full operation of which would place us out of the danger of any blockade, for we have within our borders almost everything else in abundance for maintaining a comfortable existence and, if necessary, prosecuting an effective war of defence.

In the matter of producing ammonia by the direct union of nitrogen and hydrogen, preliminary investigations are due to Cavendish. The remark suggests that a kindred key process, the direct union of sulphur dioxide and oxygen under the catalytic action of platinum was first pointed out by Davy. In these cases we have two important data in direct synthesis initiated by British workers and made vital industries by German methods. The basis of these successes of the German chemists and chemical engineers is the equal cultivation of theory and practice and the patient pursuit of the investigations. Other nations have been much too influenced by the spell of immediate practical results. Considerable information is given as to the operation of the Haber process which seems to be one that is to dominate the nitrogen-fixation industry. This was operated extensively during the late war and was, indeed, one of Germany's most important assets, for without a supply of nitrates the manufacture of the high explosives would have been impossible. It has been authoritatively stated in a German work recently published that from the time the British blockade was proclaimed, shortly after the declaration of war, until January of the following year, only a few thousand tons of Chile nitrates reached Germany. This shows well the value of sea-power and the foresight of Great Britain in creating a powerful navy, by which the seven seas were locked up within twenty-four hours after the war had really begun and were kept locked until the armistice.

The immense mass of data in the volume in hand defies even enumeration let alone careful valuation. The methods and merits of the volume in hand are the same as those of the preceding volume and the work now finished has no superior as a work of reference for the works chemists and chemical engineer.

HENRY LEFFMANN.

SYSTEMATIC ORGANIC CHEMISTRY. By W. M. Cumming, B.S., I. Vance Hopper, B.S., T. Sherlock Wheeler, B.S. xxii–535 pages, 8vo, 77 illustrations. New York, D. Van Nostrand Company, 1924. Price, $6.

Multiple authorship in books and journal contributions seems to be on the increase, possibly due to the increasing intensive specialism, which is limiting the supply of the so-called "all-around men." The three authors of the work in hand have had a large experience in research work and actual manufacturing operations, and two of them are also teachers. From such ability and practice we may expect a commendable volume and examination does not disappoint the expectation. The book, however, belongs to a class which has been very largely exploited of recent years, and partakes in the main of the general character of the class. Up-to-date methods have, of course, been chosen, and some of these

have been developed in the authors' laboratories. In the preface, the authors discuss a point which is comparatively little regarded, namely, that, in the modern system of teaching, the student is apt to get the idea that there are two phases of organic chemistry, that of the lecture-room and that of the laboratory. It is claimed that owing to this defective method, a student may know the general method for the preparation of acid anhydrides and yet be quite at sea when asked to prepare some other anhydride than acetic. It is doubtful if this lack of resourcefulness—which is common in all laboratories of applied science—is due to the lack of coördination of the lecture and laboratory courses. In American schools it is principally ascribable to the inability of the student to think outside of his immediate studies. Possibly European schools have a more severe standard for admission, so that certificates or diplomas from the institutions from which the students come to take science courses are practically uniform in their grading, hence the teacher of science can assume a certain degree of knowledge on the part of every member of the class. In American schools, however, no such uniformity exists. Two students may come with full high school certificates, but one may be from a well-organized city school in which the professor of chemistry is a university graduate with some practical laboratory experience, while the other applicant may come from a rural school in which several sciences are taught by one person, and in which the textbook has been adopted by a school-board under personal or political pressure by publisher or author.

Only favorable criticism is to be made of the book in hand. It contains a very large amount of information clearly and carefully set forth. In selecting from the vast mass of synthetic procedures in organic chemistry, each teacher and author will have a personal leaning, and hence each textbook will give prominence to certain lines of work. Considerable space is given to manipulations and attention is often drawn to the dangers of certain operations. It might be possible to give a rather large list of procedures in which no appreciable danger is involved.

The proof-reading in a book of this type is very difficult, and errors are sure to occur. Notwithstanding the joint control of three persons, the list of errors is rather large. A notable feature is the introduction of several typical methods of electrolytic transformation.

The book covers a wide field and will probably require considerable time for complete employment of it in the instruction laboratory, but the student who goes through it carefully and thoughtfully will gain an excellent idea of the methods and results of modern organic chemistry. HENRY LEFFMANN.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS. Report No. 177, The Effect of Slipstream Obstructions on Air Propellers, by E. P. Lesley and B. M. Woods. 24 pages, illustrations, quarto. Washington, Government Printing Office, 1924.

This report describes an investigation to determine the effect of slipstream obstructions on air propellers. The screw propeller on an airplane is usually placed near other objects, and hence its performance may be modified by them. Results of tests on propellers free from slipstream obstructions, both fore and aft, are therefore subject to correction for the effect of such obstruction, and

the purpose of the investigation herein described was to determine the effect upon the thrust and torque coefficients and efficiency, for previously tested air propellers, of obstructions placed in the slipstream; it being realized that such previous tests had been conducted under somewhat ideal conditions that are impracticable of realization in flight.

At the start it was planned to use obstructions representative of the nose of the fuselage, of radiators, or of other parts of an airplane structure, but a consideration of the wide variety of forms thus defined led to the selection of simple geometrical forms for the initial investigation. Such forms offered the advantage of easy exact reproduction at another time, or in other laboratories, and it was believed that the effects of obstructions usually encountered might be deduced or surmised from those of the ones chosen.

Report No. 183, The Analysis of Free Flight Propeller Tests and Its Application to Design, by Max M. Munk. 12 pages, illustrations, quarto. Washington, Government Printing Office, 1924.

A description is given of a new and useful method suitable for the design of propellers and for the interpretation of tests with propellers.

The fictitious slipstream velocity computed from the absorbed horsepower is plotted against the relative slip velocity. It is discussed in detail how this velocity is obtained, interpreted, and used. The methods are then illustrated by applying them to model tests and to free flight tests with actual propellers.

Report No. 184, The Aerodynamic Forces on Airship Hulls, by Max M. Munk. 20 pages, illustrations, quarto. Washington, Government Printing Office, 1924.

An account is here given of the new method for making computations in connection with the study of rigid airships, which are used in the investigation of Navy's ZR-1 by the special subcommittee of the National Advisory Committee for Aeronautics appointed for this purpose. It presents the general theory of the air forces on airship hulls of the type mentioned, and an attempt has been made to develop the results from the very fundamentals of mechanics, without reference to some of the modern highly developed conceptions, which may not yet be thoroughly known to a reader uninitiated into modern aerodynamics, and which may perhaps for all times remain restricted to a small number of specialists.

DURÉES PHYSIQUES INDEPENDANTS DES DIMENSIONS SPATIALES. Par Michel Petrovitch, University of Belgrade. Pamphlet, 28 pages, 8vo. Paris, Albert Blanchard, 1924.

This is a discussion of time in its relation to space and in connection with the modern development of the theory of relativity. The text involves very profound reasoning. The author sums up by quoting with approval a remark of Eddington, "We have discovered a strange footprint on the shore of the unknown. To explain its origin we have built theory on theory, each more profound and more ingenious than its predecessor. We have finally materialized the being that made the mark and find that it is ourself."

PUBLICATIONS RECEIVED.

Chemisch-Technische Vorschriften. Ein Handbuch der Chemischen Technologie von Dr. Otto Lange. iv Band. Düngemittel (Sprengstoffe) Futtermittel, Lebensmittel. 750 pages, 8vo. Leipzig, Otto Spamer, 1924. Price, in paper, 45 gold marks; bound, 50 gold marks.

Chemical Synonyms and Trade Names. A dictionary and commercial handbook by William Gardner. 271 pages, 8vo. London, Crosby Lockwood and Son, 1924.

Encyclopédie d' Électricité Industrielle. Bibliothèque des Annales des Postes, Télégraphes et Téléphones. Appareils et installations téléphoniques par E. Reynaud-Bonin. 487 pages, illustrations, plates, 8vo. Paris, J. B. Baillière et Fils, 1924. Price, in paper, 50 Francs.

Oil Engines. By A. L. Bird. 281 pages, illustrations, plates, 8vo. New York, E. P. Dutton and Company, 1923. Price, $5.

The Foundations of Einstein's Theory of Gravitation. By Erwin Freundlich with a preface by Albert Einstein. Translated from the fourth German edition, with two essays, by Henry L. Brose, with an introduction by H. H. Turner. 140 pages, illustrations, 12mo. New York, E. P. Dutton and Company. Price, $2.50.

Systematic Organic Chemistry. Modern methods of preparation and estimation by William M. Cumming, J. Vance Hopper and T. Sherlock Wheeler. 535 pages, illustrations, 8vo. New York, D. Van Nostrand Company, 1924. Price, $6.

A Text-book of Inorganic Chemistry. Edited by J. Newton Friend. Vol. II. The alkali-metals and their congeners by A. Jamieson Walker. 379 pages, illustrations, 8vo. London, Charles Griffin and Company; Philadelphia, J. B. Lippincott Company, 1924.

The Theory of Relativity. Three lectures for chemists by Erwin Freundlich, translated by Henry L. Brose, with an introduction by Viscount Haldane. 98 pages, 12mo. New York, E. P. Dutton and Company. Price, $2.

Problems in General Physics for College Courses. By Martin Masius. Second edition, revised and enlarged. 154 pages, illustrations, 12mo. Philadelphia, P. Blakiston's Son and Company. Price, $2.

Statique et Résistance des Matériaux. Par Paul Montel. 273 pages, illustrations, 8vo. Paris, Gauthier-Villars et Cie., 1924. Price, in paper, 30 Francs.

Aide-Mémoire de Céramique Industrielle. Par Alix Cornille. 184 pages, illustrations, 8vo. Paris, "Revue des Matériaux de Construction et de Travaux Publics, 1924.

Électricité Atmosphérique. Par B. Chauveau. Troisième fascicule. Généralités sur les ions, l'ionisation et la radioactivé. La conductibilité et l'ionisation de l'atmosphère. 240 pages, illustrations, 8vo. Paris, Octave Doin, 1924. Price, 24 Francs.

U. S. Department of Commerce, Bureau of Standards, Circular No. 74.

Radio instruments and measurements. Second edition. 345 pages, illustrations, plates, 8vo. Washington, Government Printing Office, 1924. Price, sixty cents.

Bibliographie de la Relativité. Suivie d'un appendice sur les determinants a plus de deux dimensions le calcul des variations les séries trigonométriques et l'azéotropisme par Maurice Lecat avec la collaboration de Mme. M. Lecat-Pierlot. 290–47 pages, 8vo. Bruxelles, Maurice Lamartin, 1924.

Heat Dissipation from the Surfaces of Pipes and Cylinders in an Air Current. A. H. GIBSON. (*Phil. Mag.*, Feb., 1924.)—A series of pipes of different metals was wound into spirals and exposed in the wind-tunnel of the Royal Aircraft Establishment, Farnsborough, to air moving with speeds up to 44.5 miles per hour. Hot water was sent through the pipes and from its difference of temperature upon entrance and at leaving the coils the heat loss was calculated. It was found that the heat loss in a cooling wind for pipes ranging in temperatures from 40° to 100° C. is proportional to the nth power of the difference of temperature between air and pipe, and that n has the value, 1.02. If the heat removed be taken to be as the nth power of the velocity of the wind, then n has values ranging from .50 for small pipes and slow winds to .72 for large pipes and winds of about 40 miles per hour. The effect of the character of the surfaces of the pipes was not as marked as would have been expected. Sand-blasting surfaces of aluminum and of copper raised the rate of heat loss in still air and in the case of copper in a 35 m.p.h. wind, but lowered it with aluminum in the same wind. Coating the coil with dull, black spirit paint in every instance increased the heat loss, though the percentage of change was considerably greater in quiet air than in a wind. With a wind of 35 m.p.h. this paint raised the rate of heat dissipation by 4.5 per cent., which was largely due to increased loss by radiation. G. F. S.

Determination of Small Quantities of Lead.—In a paper presented in November last to the British Society of Public Analysts and published in *The Analyst* (1924, **49**, 124), Dr. J. C. Thresh gives procedures for determining minute amounts of lead in potable water and urine. It depends on the production of colloidal lead sulphide and comparison with standards. The colloidal condition is obtained by precipitating the metal in the presence of an acetic acid gelatin, prepared as follows:

Acetic acid, 36 per cent.	27.0 c.c.
Water	730 c.c.
Gelatin	0.1 grm.

The gelatin should be of good quality such as that used for bacteriologic purposes. It is to be dissolved in the mixture of acid and water at a temperature below the boiling point. About 1 c.c. of the solution will be needed for 50 c.c. of water. The color of many surface waters interferes with the test. Results with other waters

and test-analyses were found to differ, and Doctor Thresh found that water distilled in glass vessels was uniform, but that distilled in tinned copper contained minute amounts of the latter metal, sufficient to interfere. One part of copper in 100,000,000 of water has a distinct interfering action, the sulphide being much paler than lead sulphide. In a natural water from South Wales copper to this dilution was found. If metallic tin is immersed in the copper-containing water, the latter metal seems to be removed and the sample gives a normal reaction. If a silver spoon is left over night in water a condition is produced which has an interfering effect similar to that of minute amounts of copper. (Is it possible that this may be due to copper, as all commercial silver articles contain considerable amounts of the baser metal? The recent researches on dezincing of brass have shown that in selective corrosions the whole alloy may dissolve and the less positive metal be redeposited. *Note by abstractor.*)

For the determination of lead in urine, Thresh advises the following process: At least 100 c.c. of the sample is heated nearly to boiling and normal sodium carbonate solution added until the liquid becomes turbid. The precipitate flocculates on gentle shaking. Generally 1 c.c. of the sodium carbonate solution will suffice. The liquid is allowed to stand until clear. This can be poured off or passed through a small filter, upon which the whole precipitate is collected, washed with distilled water, made alkaline with sodium carbonate and finally dissolved in about 2 c.c. of acetic acid (36 per cent.) mixed with 12 c.c. of hot water. The total filtrate need not exceed 25 c.c. To this filtrate is added 0.5 c.c. of the acetic acid gelatin and hydrogen sulphide. The color should be compared with standard in the usual manner. It is important that no results should be accepted until the analyst has tested out the procedure with lead-free samples. Thresh found in the course of this investigation that common filter papers may contain lead. It is, therefore, necessary to use paper thoroughly exhausted with acid. Attempts to determine lead in beer and cider were at first prevented by the large proportion of coloring matter thrown down, but by using precipitated chalk instead of sodium carbonate the precipitate is very pale and the method promises to be satisfactory on further experiment.

The original paper on the method appeared in *The Analyst* (1921, **46**, 270). (In giving this reference, Thresh has inadvertently given the volume as **45**, but the correct reference has been verified.) The method is well worth trial, but it will be necessary for the analyst to make a number of test-analyses to get the proper technique.

H. L.

PRESS OF
J. B. LIPPINCOTT COMPANY
PHILADELPHIA

INDEX TO VOL. 197.

Atom, Structure of the (Swann), 293

BARDWELL, D. C., and S. C. LIND: The coloring and thermophosphorescence produced in transparent minerals and gems by radium radiation (Correspondence), 253

BAZZONI, C. B., and C. T. CHU: Soft X-rays from tungsten, 183

BAZZONI, C. B., and A. T. WALDIE: Impact effects in nitrogen and nitric oxide, 57

BERG, ESKIL: Pressure, superheat, steam extraction and reheating as affecting power plant economy, 727

BINGHAM, EUGENE C.: Plasticity and elasticity, 99

Biographical note, Walton Clark, 284; W. C. L. Eglin, 286

BLIZARD, JOHN: The terminal velocity of particles of powdered coal falling in air or other viscous fluid, 199

BREIT, G.: A singular case of electron tube oscillations, 355

BOOK REVIEWS:

Allen's commercial organic analysis, ed. 5, vol. 1, 577

Armbruster, H. W.: Arsenic, calcium arsenate and the boll-weevil, 430

Bavink, B.: L'atomistique, 427

Calcott, W. S., J. C. Whetzel and H. F. Whittaker: Corrosion tests and materials of construction for chemical engineering apparati, 574

Colloid symposium monograph, 431

Cumming, W. M., I. V. Hopper and T. S. Wheeler: Systematic organic chemistry, 859

Eastman Kodak Company Research Laboratory: Abridged scientific publications, vol. 6, 720

Fajans, K.: Radioactivity, 428

Gardner, W.: Chemical synonyms and trade names, 857

Gibbs, W. E.: Clouds and smokes, 575

Goodrich, W. F.: Pulverized fuel, 578

Gregory, Sir Richard: The vault of heaven, 138

Groggins, P. H.: Aniline and its derivatives, 723

Hering, D. W.: Foibles and fallacies of science, 719

Hildebrand, J. H.: Solubility, 722

Hughes, W. E.: Modern electroplating, 579

Humphreys, W. J.: Weather proverbs and paradoxes, 139

Lange, O.: Chemisch-technische Vorschriften, ed. 3, vol. 3, 432; vol. 4, 858

Luttringer, A. D.: Aide-mémoire pour l'industrie caoutchouc et des matières plastiques, 140

Molinari, E.: Treatise on general and industrial organic chemistry, ed. 2, part II, 580

National Advisory Committee for Aeronautics, Reports: No. 174, 433; No. 175, 141; No. 176, 581; No. 177, 860; No. 178, 433; No. 179, 434; No. 182, 725; No. 183, 861; No. 184, 861

Petrovitch, M.: Durées physiques independents des dimensions spatiales, 861

Philadelphia College of Pharmacy, Popular science talks, 725

Popoff, S.: Quantitative analysis, 430

Svedberg, T.: Colloid chemistry, 717

Svenson, C. L.: The art of lettering, 721

Tressler, D. K.: Marine products of commerce, 425

Weber, G. A.: The patent office, 721
CHU, C. T., and C. B. BAZZONI: Soft X-rays from tungsten, 183
CLARK, WALTON: Biographical note, 284
Coal, The terminal velocity of particles of powdered, falling in air (Blizard), 199
Coast and Geodetic Survey, The tidal and current work of the (Rude), 73, 217
Coloring and thermophosphorescence produced in transparent minerals and gems by radium radiation (Correspondence) (Lind and Bardwell), 253
CURRENT TOPICS: 56, 72, 97, 98, 116, 124, 126, 143, 182, 198, 207, 215, 216, 251, 252, 253, 254, 262, 265, 268, 272, 291, 292, 321, 358, 400, 404, 407, 408, 410, 436, 504, 516, 525, 526, 539, 555, 557, 561, 565, 601, 621, 627, 628, 666, 690, 696, 698, 700, 704, 726, 739, 740, 756, 786, 820, 824, 827, 828, 838, 846, 863, 864
Current work of the Coast and Geodetic Survey (Rude), 73, 217
DAVEY, WHEELER P.: Radiation, 439, 629
Eastman Kodak Company Research Laboratory Notes, 263, 405, 557, 697
EGLIN, W. C. L.: Biographical note, 286
Electron mobilities, The effect of variable, on the formation of negative ions in air (Loeb), 45
Electron tube oscillations, A singular case of (Breit), 355
Ferromagnetism (McKeehan), 583, 757
FORSYTHE, W. E.: The optical pyrometer as a brightness photometer, 517
FRANKLIN INSTITUTE:
Board of Managers, Minutes of the annual meeting, January 23, 1924, 415
Committee on Science and the Arts, Abstract of proceedings: December 5, 1923, 132; January 2, 1924, 288; February 6, 1924, 421; March 5, 1924, 568; April 2, 1924, 708; May 7, 1924, 848
Library Notes, 133, 422, 569, 711, 849
Membership Notes, 133, 290, 421, 568, 709, 849
Stated meetings, Proceedings: December 19, 1923, 131; January 16, 1924, 273; February 20, 1924, 419; March 19, 1924, 567; April 16, 1924, 705; May 21, 1924, 847
HACHEY, H. B.: The rotation of melting ice suspended in benzine, 825
HAYES, HARVEY C.: Measuring ocean depths by acoustical methods, 323
High-voltage phenomena (Peek), 1
HUMPHREYS, W. J.: The relation of sound velocity to height, 821
Hydrogen-ion concentration in relation to animal and plant growth (Leffmann), 741
Ice, melting, The rotation of, suspended in benzine (Hachey), 825
Impact effects in nitrogen and nitric oxide (Bazzoni and Waldie), 57
Interrelations of modern physics and modern psychology (Troland), 479
Iron. Ferromagnetism and its dependence upon chemical, thermal and mechanical conditions (McKeehan), 583, 757
IVES, HERBERT E.: A primary standard of light, following the proposal of Waidner and Burgess, 147, 359
KENNELLY, A. E.: The reluctivity of the recently discovered magnetic metal, permalloy, 623
LEFFMANN, HENRY: Hydrogen-ion concentration in relation to animal and plant growth, 741
Light, Primary standard of (Ives), 147, 359
LIND, S. C., and D. C. BARDWELL: The coloring and thermophosphorescence produced in transparent minerals and gems by radium radiation (Correspondence), 253

LOEB, LEONARD B.: The effect of variable electron mobilities on the formation of negative ions in air, 45
McKEEHAN, L. W.: Ferromagnetism and its dependence upon chemical, thermal and mechanical conditions, 583, 757
MAIZLISH, ISRAEL: The scattering of X-rays, 667
Measuring ocean depths by acoustical methods (Hayes), 323
Minerals and gems, transparent, Coloring and thermophosphorescence produced in, by radium radiation (Correspondence) (Lind and Bardwell), 253
Negative ions in air, The effect of variable electron mobilities on the formation of (Loeb), 45
Nitrogen and nitric oxide, Impact effects in (Bazzoni and Waldie), 57
Ocean depths, Measuring (Hayes), 323
Optical properties of fluorescent rhodamin B (Poritsky), 527
Optical pyrometer as a brightness photometer (Forsythe), 517
Oscillations and resonance in systems of parallel connected synchronous machines (Putman), 603, 787
PEEK, F. W.: High-voltage phenomena, 1
Permalloy (Kennelly), 623
Plasticity and elasticity (Bingham), 99
Plate with a circular hole, On stresses in a (Timoshenko), 505
Photometer, brightness, The optical pyrometer as a (Forsythe), 517
Physics, modern, and modern psychology, The interrelations of (Troland), 479
PORITZKY, A.: Some optical properties of fluorescent rhodamin B, 527
Power plant economy, Pressure, superheat, steam extraction and reheating as affecting (Berg), 727
Pressure, superheat, steam extraction and reheating as affecting power plant economy (Berg), 727
Primary standard of light (Ives), 147, 359
Protractor, Vector-ruling (Zahm), 209
Psychology, modern, The interrelations of modern physics and (Troland), 479
Publications received, 141, 290, 434, 581, 726, 862
PUTMAN, H. V.: Oscillations and resonance in systems of parallel connected synchronous machines, 603, 787
Pyrometer, The optical, as a brightness photometer (Forsythe), 517
Radiation (Davey), 439, 629
Reluctivity of the recently discovered magnetic metal, permalloy (Kennelly), 623
Rhodamin B, Optical properties of fluorescent (Poritsky), 527
RUDE, G. T.: The tidal and current work of the Coast and Geodetic Survey, 73, 217
Scattering of X-rays (Maizlish), 667
Sound velocity, The relation of, to height (Humphreys), 821
Steam extraction, pressure, superheat and reheating as affecting power plant economy (Berg), 727
Stresses in a plate with a circular hole (Timoshenko), 505
Structure of the atom (Swann), 293
Superheat, pressure, steam extraction and reheating as affecting power plant economy (Berg), 727
SWANN, W. F. G.: The structure of the atom, 293
Synchronous machines, parallel connected, Oscillations and resonance in systems of (Putman), 603, 787
Tidal and current work of the Coast and Geodetic Survey (Rude), 73, 217
TIMOSHENKO, S.: On stresses in a plate with a circular hole, 505

TROLAND, L. T.: The interrelations of modern physics and modern psychology, 479
Tungsten, Soft X-rays from (Bazzoni and Chu), 183
U. S. Bureau of Chemistry Notes, 125, 267, 409, 559, 699, 839
U. S. Bureau of Mines Notes, 127, 269, 411, 563, 701, 841
U. S. Bureau of Standards Notes, 117, 255, 401, 541, 691, 829
Vector-ruling protractor (Zahm), 209
Velocity, The terminal, of particles of powdered coal falling in air (Blizard), 199
WALDIE, A. T., and C. B. BAZZONI: Impact effects in nitrogen and nitric oxide, 57
X-rays, The scattering of (Maizlish), 667
X-rays, Soft, from tungsten (Bazzoni and Chu), 183
ZAHM, A. F.: The vector-ruling protractor, 209